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A Comparison of Two Methods of Reducing Truncation Error

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A COMPARISON OF TWO METHODS OF REDUCING TRUNCATION ERROR

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TECHNIQUES DEVELOPMENT LABORATORY

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A COMPARISON OF TWO METHODS OF REDUCING TRUNCATION ERROR

Robert J. Bermowitz

ABSTRACT

The comparative reduction of truncation error by shrinking the space scale and using a higher order approximation of the derivative is examined. It is found that for wavelengths less than about 2000 km., use of the second order approximation with a grid interval of 190.5 km. results in a greater reduction of truncation error than a fourth order approximation with a grid interval of 381 km.

Two fourth order approximations are also compared and found to give similar results.

Considering the effects of truncation error alone, it is inferred that the shortest wavelength whose movement can be predicted with about 90% accuracy, with use of a fourth order approximation, is approximately 4.5 times the grid interval being used in the computations.

INTRODUCTION

Recent research in short range, operational numerical forecasting has emphasized the prediction of the relatively short waves. For this purpose, numerical models are being solved on grids with space scales smaller than have been previously used. The efforts of Howcroft [4], Bushby and Timpson [1], Hill [3], Wang et. al. [9], and Gerrity and McPherson [2] can be cited as examples.

One obvious reason for diminishing the space scale is to reduce the error inherent in approximating derivatives by finite differences. This truncation error, which is most serious for the higher frequency waves, results in phase speeds lower than those actually observed for a given forecast period. In the interest of improving the quality of forecasts, it would be desirable to reduce the truncation error as much as possible. There are other means of decreasing the truncation error which involve the use of higher order approximations of the derivative, e.g. Miyakoda [6] and Shuman and Vanderman [8].

The purpose of this paper is to compare the effectiveness of reducing truncation error by diminishing the grid interval and by using higher order approximations of the derivative. In the process, several operationally oriented finite difference approximations are compared. Also, an inference is made regarding the shortest wavelength which can be predicted with an acceptable amount of truncation error.

EXPERIMENTAL DESIGN

If the vorticity equation is approximated by finite differences, the advection term represented by $J(\psi,\xi)$, contains the largest truncation error (Miyakoda [5]). Here $\psi(x,y)$ is the stream function, ξ is the relative vorticity (the Coriolis parameter has been omitted) and J is the Jacobian. This experiment attempts to compare values of $J(\psi,\xi)$ computed by various finite difference approximations, denoted by $J_c(\psi,\xi)$, with analytic values of $J(\psi,\xi)$. As a measure of accuracy of $J_c(\psi,\xi)$, the response R, or $J_c(\psi,\xi)/J(\psi,\xi)$, is computed for each of the approximations.

Although quite arbitrary, and certainly not critical to the results of this study, an approximate stream function field at 500 mb. is used as the analytic function. Consequently, $\psi(x,y)$ is approximated as consisting of a constant zonal current upon which is superimposed a two-dimensional wave pattern as follows:

$$\psi(x,y) = A + B\left(1 - \frac{y}{35d}\right) + C \sin\frac{S\pi x}{35d} \sin\frac{S\pi y}{35d} . \qquad (1)$$

Here x is the eastward direction,

y is the northward direction,

d is the grid interval,

A, B, C are constants given the values 47×10^7, 7×10^7 and $.5 \times 10^7$, respectively, and

S is a parameter proportional to wave number, and takes on values between 1 and 18.

The finite difference approximations of the derivative that are used to calculate $J_c(\psi,\xi)$ are illustrated schematically in Figure 1 for the derivative in the x-direction. Figure 1 (a) is the usual 3 point second order approximation of the derivative. Figures 1 (b) and 1 (c), the 5 and 25 point operators, respectively, are fourth order approximations. The 5 point operator which has been examined by Miyakoda [6, 7], is used operationally at the Japan Meteorological Agency. The 25 point operator, discussed by Shuman and Vanderman [8], has been used operationally at the National Meteorological Center since 1963. It consists of the 5 point operator, and a 5 point smoother which is applied in the horizontal dimension other than in which the derivative is computed.

$J_c(\psi,\xi)$ and R are computed for two values of grid interval, 381 km. on an 18 x 18 grid, and 190.5 km. on a 35 x 35 grid. The value of 381 km. is used operationally at the Weather Bureau's National Meteorological Center. $J(\psi,\xi)$ at every point on the 18 x 18 grid has the same value as $J(\psi,\xi)$ at every other point on the 35 x 35 grid.

$$\frac{1}{2d}\begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

(a)

$$\frac{1}{12d}\begin{bmatrix} 1 & -8 & 0 & 8 & -1 \end{bmatrix}$$

(b)

$$\frac{1}{432d}\begin{bmatrix} -1 & 8 & 0 & -8 & 1 \\ 4 & -32 & 0 & 32 & -4 \\ 30 & -240 & 0 & 240 & -30 \\ 4 & -32 & 0 & 32 & -4 \\ -1 & 8 & 0 & -8 & 1 \end{bmatrix}$$

(c)

Figure 1. Finite difference approximations of the derivative in the x-direction, (a) 3 point, (b) 5 point, (c) 25 point. The weights at each point of (a), (b), (c) are to be multiplied by the preceding coefficient.

RESULTS AND CONCLUSIONS

The results of the experiment are shown in Figure 2, where the response is plotted as a function of S and wavelength. The dashed and solid lines represent the responses of the finite difference approximations calculated for grid intervals of 190.5 km. and 381 km., respectively. For the analytic function used here, it has been found from the computations that the responses of the three operators are not functions of x or y. Thus the values shown in Figure 2 can be thought of as the response at any point, or the average response of all points.

The responses of the operators shown in Figure 2 are very similar to those obtained by Shuman and Vanderman [8]. The superiority of the fourth order approximation of the derivative in comparison to the second order approximation is readily apparent. Although not to the same degree, a similar statement can be made for the 5 point operator when compared to the 25 point operator, at least for the analytic function used here. If the superimposed component of the flow represented by the third term on the right hand side of equation (1) was a function of x only or y only, then the response using the 25 point operator would be the same as that of the 5 point operator shown in Figure 2. This has been verified experimentally. In this case, the derivatives as computed by the 5 point operator would be functions of x only or y only; consequently, the application of a one-dimensional smoothing operator in the dimension other than in which the derivatives are computed would not change the 5 point result.

It should be noted that at the Techniques Development Laboratory, several hemispheric, barotropic, 500 mb. forecasts with real data and a space scale of 190.5 km., have shown little or no difference between the 5 and 25 point operators. However, experience with these operators has indicated that if computational instability is apt to occur, it is more likely to happen at an earlier time step with the 5 than with the 25 point operator. The smoother in the latter may account for this difference. On the other hand, an additional difference between the two is the time required for computing $J_c(\psi,\xi)$. The method suggested by Shuman and Vanderman [8] using the 25 point operator results in approximately twice as many arithmetic calculations as that resulting from direct application of the 5 point operator to each of the derivatives contained in $J(\psi,\xi)$. It would thus seem more economical to use the 5 point operator, since the forecasts obtained by using 5 and 25 point operators on real data are very nearly the same.

Of further interest in Figure 2 is the comparative reduction of truncation error obtained by shrinking the space scale and by using a higher order approximation of the derivative. Nearly the same reduction of truncation error is obtained for wavelengths greater than about 2000 km. with use of the 3 point operator and a grid interval of 190.5 km., as with the 5 point operator and a space scale of 381 km. For wavelengths less than about 2000 km., use of the former results in a considerably better response. The important point is that there is a wavelength below which truncation error will remain tolerable only by reducing the space scale. However, no matter how small the space scale is made, the response for all operators will approach zero for wavelengths nearing twice the grid interval.

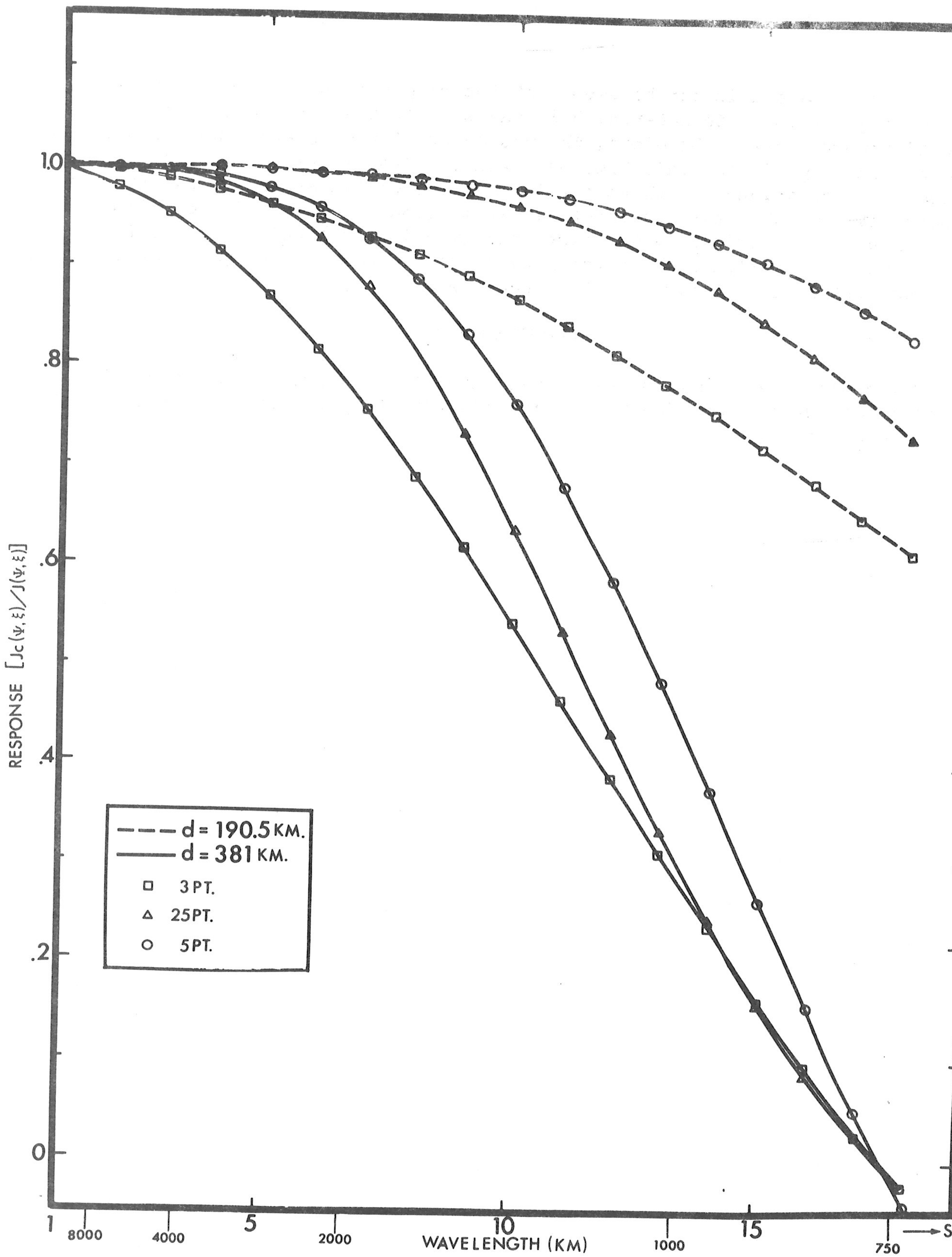

Figure 2. The responses of the finite difference approximations shown in figure 1 plotted as a function of S and wavelength. The dashed and solid lines represent responses calculated for grid intervals of 190.5 km. and 381 km., respectively.

From Figure 2 it can be seen that the response for a given operator at wavelength λ and grid interval δ is the same as that at wavelength $\lambda/2$ and grid interval $\delta/2$. Therefore, the responses of the several operators can be obtained at grid intervals $\delta/4$, $\delta/8$, etc.. With use of the 5 point operator, and with acceptance of 10% as a tolerable truncation error, it can thus be seen that the shortest wavelength whose movement can be predicted with 90% accuracy is approximately 4.5 times the grid interval being used in the computations. Effects which can reduce the forecast accuracy, other than that of truncation error, have not been considered here.

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