WEATHER BUREAU
Systems Development Office
Techniques Development Laboratory
Silver Spring, Md.

JUNE 1967

A Survey of Studies of Aerological Network Requirements

M. A. ALAKA

Technical Memorandum WBTM TDL-9

U.S. DEPARTMENT OF COMMERCE / ENVIRONMENTAL SCIENCE SERVICES ADMINISTRATION

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A SURVEY OF STUDIES OF AEROLOGICAL NETWORK REQUIREMENTS

M. A. Alaka

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June 1967

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M. A. ALAKA

1. INTRODUCTION

The problem of determining the optimum spacing of aerological networks dates from the invention of the radiosonde. We learn from the annals of the International Meteorological Organization (IMO), the predecessor of the World Meteorological Organization (WMO), that as far back as 1953 the Conference of Directors, which met in Warsaw, was informed that the scientific institutes of USSR were engaged in establishing criteria for the proper distribution of upper-air observing stations.

The problem grew in importance during the post-war years with the greatly increased availability of radiosondes and greater demand for aviation forecasts over long international routes. It was inevitable, owing to the international scope of the problem, that it should engage the interest of IMO. At its Paris session in 1946, the Commission for Synoptic Weather Information made the first quantitative evaluation of the global network requirements. The criteria for these requirements (see Table 1), however, were derived somewhat arbitrarily from limited experience in temperate latitudes rather than on the basis of a rigorous, systematic study of the problem.

The growing demand for increased upper air observations was, at that time, somewhat tempered by the fact that field forecasters were already confronted with a vast amount of data only a small fraction of which they could assimilate and utilize. During the past decade two parallel developments have imparted a new urgency to the problems of providing more upper air meteorological observations and, because of the great expense involved, of determining as accurately as possible the requirements for these observations.

First, the advent and increasing availability of more and more powerful computers have vastly enhanced our ability to process and utilize meteorological data.

Secondly, the development of increasingly sophisticated dynamical weather prediction models have greatly improved our potential of improving both the accuracy and range of weather forecasts. But this potential is marred by the great sensitivity of dynamic prediction techniques to inaccuracies in the definition of the initial state of the atmosphere. Newton (1954) has described some of the problems which can arise from errors in the description of the initial state. Best (1956) and Berggren (1957) have shown that numerical forecasts made from analyses of the same initial data by different forecasters can lead to very different results, especially after 48 hours.

It is therefore not surprising that the more serious attempts to define criteria for the optimum distribution of upper air stations are of recent date.

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*This paper was prepared for the Systems Development Office study on Design of the Data Acquisition Subsystem (DDAS) and was included as part of the final report of Task Group IV-3a, "Determining Theoretical Relationships of Upper Air Networks."

TABLE 1

Some Estimates of Suitable Station Spacing for Aerological Networks

Source	Relevant to	Spacing (Km)
Commission for Synoptic Weather Information (1946)	Basic network	300
	Thinly populated area	1000
	Oceans	1000
Working Group, Commission for Aerology, WMO (1960)	Stations in temperate latitudes reporting both pressure and wind	500-600
	Stations reporting wind only	300-350
Working Group, Commission for Synoptic Meteorology, WMO (1960)	Temperate Zone	650
	Polar regions	1100
	Tropical regions	1600
Weather Bureau Panel (1963)	Mainly middle latitudes	1000-1300
Dobryshman (1963)	Temperate Zone	300
	Polar regions	600-800
	Tropical regions	1200-1500
Jess (1960)	Mainly middle latitudes	900

In the following pages the more important of these studies are reviewed.

2. THEORETICAL STUDIES

2.1. Errors of Analysis - The general properties of the initial condition errors may be deduced theoretically by regarding the meteorological field to be analyzed as a sum of harmonic components. A given network of stations can resolve only those harmonic components whose scale is larger than the spacing of the stations. The unresolved smaller-scale components will appear in the measured values much like random noise which is erroneously interpreted as part of the larger-scale components. The effect of this "aliasing" is similar to that of random observation errors and adds to the uncertainty in determining the larger-scale components. This uncertainty, the "noise-level," may be measured by the total power (variance) of all the components of the field whose scale is smaller than the spacing of the observing stations. A network of stations would be adequate if the total power of the noise level were a small fraction of the total variance of the field.

An attempt to evaluate network requirements on the above basis was made by a Working Group on Networks established by the Commission for Aerology (CAe) of WMO. In its report, published as WMO Technical Note No. 29 (Eliassen et al., 1960), the Working Group took note of a previous study by Eliassen (1954) who found that the autocorrelation function of the pressure height fields at different levels over the British Isles in December and January had values between 0.90 and 0.95 for two points 300 km apart. If these values are representative of average atmospheric conditions and if the autocorrelation function decreases monotonically with distance up to 300 km, then a network with stations approximately this distance apart would define the initial state of the atmosphere adequately, provided the random errors of observations inherent in the analysis technique are small enough.

A more elaborate series of theoretical studies of network requirements is contained in WMO Technical Note No. 30 (Bessemoulin et al., 1960) which embodies the preliminary report of the Working Group on Networks of the Commission for Synoptic Meteorology (CSM) of WMO. The errors of analysis associated with networks of different densities are traced back to the following factors:

(a) The structure of the atmosphere itself

(b) The accuracy of measuring instruments

(c) The technique of analysis.

One contribution made by P. D. Thompson makes use of a quadratic interpolation scheme to compute the error in determining the height at a point "O" (Fig. 1) in terms of the observed heights at twelve surrounding points. The interpolation formula is:

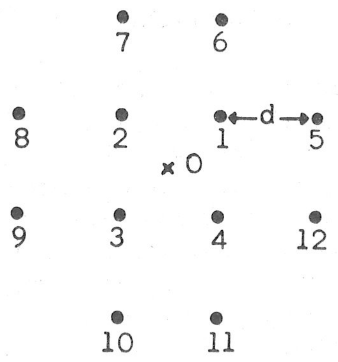
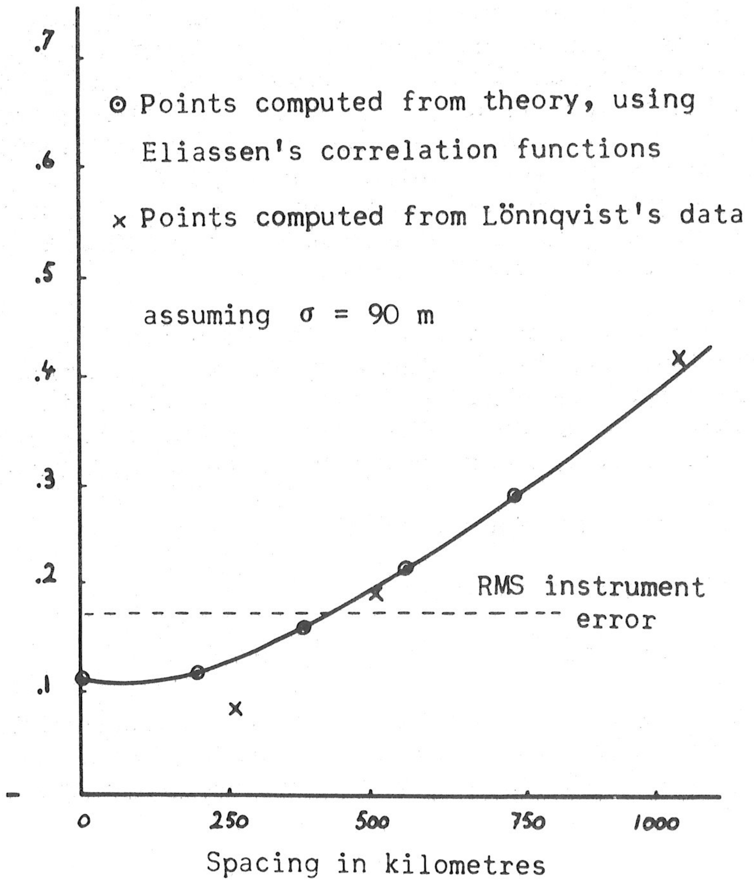


Fig. 1. Network configuration used by Thompson (Ref. 18).



Ref. 2. Variation with spacing of the root-mean-square analysis error in units of (Thompson, Ref. 18).

$$Z_o = \frac{1}{4}\sum_{i=1}^{4} z_i + \frac{1}{32}\left(2\sum_{i=1}^{4} z_i - \sum_{i=5}^{12} z_i\right) \quad (1)$$

where Z_o is the interpolated value of the deviation from average height at the point "0," and the z_i are observed deviations from average height, where the subscript denotes the point at which each value applies. The first term on the right hand side of Eq. (1) corresponds to linear interpolation with very slight smoothing, while the second term is a curvature correction term which requires that the curvature of the height field over the central square be equal to the average curvature over the whole array of points. Thompson's expression for the mean square error of interpolation is

$$\overline{E^2} = \sigma^2\left\{.398[1-4(o)] + \sum_{i=0}^{9} A_i\ r(l_i)\right\} \quad (2)$$

where σ is the standard deviation of height, r the normalized autocorrelation function for station separations, l_i and A_i are weighting factors.

Thompson's results are given in Fig. 2, which shows the variation with spacing of the root mean square analysis error in units of σ .

Another contribution to the report, made by R. Pône, utilizes an interpolation method attributed to Eliassen (1954) which has the property of minimizing the mean error. If the real value of a variable Z is measured with an error ϵ and if the measured value is designated by z, let the value Z_o at the point "0" be estimated from the linear relation:

$$Z_o = \sum_{i=1}^{n} a_i z_i \quad (3)$$

The mean square error is

$$\begin{aligned}\overline{E^2} &= \overline{(z_o - \sum_{i=1}^{n} a_i z_i} \\ &= \overline{\left[\sum_{i=0}^{n} a_i(z_i + \epsilon_i)\right]^2} \\ &= \sum_{i=1}^{n}\sum_{j=1}^{n} a_i a_j\ \overline{(z_i + \epsilon_i)\ (z_j + \epsilon_j)}\end{aligned} \quad (4)$$

or, in terms of the "noise factor," $\lambda_i^2 = \frac{\epsilon_i^2}{\sigma_i^2}$

$$\overline{E^2} = \sum_{i=1}^{n} \sum_{j=1}^{n} a_i a_j \sigma_i \sigma_j \left(\zeta_{ij} + \lambda_{ij}^2 \right) \tag{5}$$

where σ_i^2 is the variance of z_i and ζ_{ij} in the correlation between z_i and z_j.

The minimum mean square error $\overline{E_m^2}$ is obtained by differentiating the above equation with respect to a_i and setting the result equal to zero. After some manipulation one obtains the following expression

$$\overline{E_m^2} = \sigma_0^2 - \sigma_0 \sum_{j=1}^{n} a_j \sigma_j \zeta_{oj} \tag{6}$$

The error is more conveniently expressed by the dimensionless quantity

$$Q_m^2 = \frac{\overline{E_m^2}}{\sigma_0^2} = 1 - \sum_{j=1}^{n} a_j \frac{\sigma_j}{\sigma_0} \zeta_{oj} \tag{7}$$

which contains the effects of instrumental errors implicitly through a_j.

The quantity $a_j \frac{\sigma_j}{\sigma_o} = \alpha_j$ is determined from the relation

$$\sum_{j=1}^{n} \alpha_j \left(\zeta_{jk} + \lambda_{jk} \right) = \zeta_{ok} \tag{8}$$

so that the quantity Q_m^2 may be computed from the space correlation ζ_{ik} and the noise factor λ^2 without having recourse to the local values of the variances.

Such a computation was made by Pône for a triangular grid of six symmetrically arranged stations shown in Fig. 3 under the assumption that λ^2 is a constant and that the spatial correlation is isotropic and may be expressed in terms

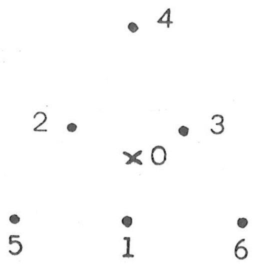


Fig. 3. Network configuration used by Pône (1960).

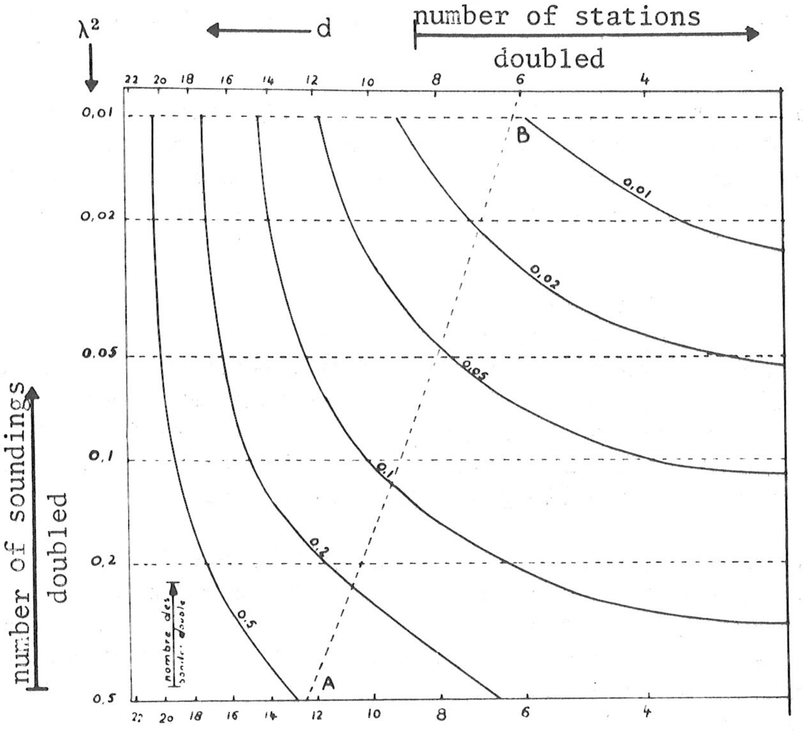


Fig. 4. The variation of the dimensionless quantity Q_m^2 with the noise factor (λ^2) and spacing d. In the area to the left of line AB, it would be more advantageous to increase station density; to the right of AB, analysis errors would be more effectively reduced by increasing precision of measurements (Pône, 1960).

of the distance d by the following relation

$$\zeta = e^{-\frac{d^2}{\mu}} \qquad (9)$$

Fig. 4 gives the computed relation of Q_m^2 in terms of log λ^2 and log d^2. Assuming that the precision of measurements may be augmented by launching several radiosondes simultaneously from the same station, the figure indicates that in the area to the left of the dashed line AB, which joins the points where the isolines of Q_m^2 have a slope of -45°, it is more advantageous to increase station density and that to the right of this line the errors in the analysis are more effectively reduced by increasing the precision of measurements. Fig. 5 compares the relative reduction of analysis error resulting from doubling the density of stations and launching two radiosondes simultaneously from each station. The figure indicates that if the network is relatively dense, error reduction is more likely to be achieved by increasing the accuracy of instruments than by adding more stations - the more so, the higher the noise level.

2.2. Amplification of Initial Errors - As mentioned above, numerical forecasts are very sensitive to initial state errors which generally grow with the forecast period.

The phenomenon of error growth has been studied from a theoretical viewpoint by Thompson (1957), who treated the problem as one relating to the stability of a disturbance superposed on a basic flow. Thompson found that the increase of the root-mean-square wind error in predictions over periods of a few days depends on:

(a) The period of the forecast
(b) The initial RMS vector wind error
(c) The difference between the characteristic scale of the initial error field and the scale of fluctuation in the true initial flow pattern
(d) The area average of the vertical wind shear between 250 mb and 750 mb
(e) The RMS vector deviation of the wind at about 500 mb from its area average
(f) The average static stability of the atmosphere.

The above results show that network density affects the accuracy of forecasts in a dual manner, since it determines both the magnitude of the initial analysis errors and the growth of these errors with time. It is conceivable that a network which is dense enough to produce an analysis with comparatively small initial errors may yet be inadequate to insure a sufficiently slow rate of growth of these errors.

2.3. Maximum Station Spacing - On the basis of Eliassen's findings, mentioned above, concerning the autocorrelation function in the vicinity of Manchester, and of some experiments by Best (1956) on the growth of initial analysis errors with time, the CAe Working Group on Networks (Eliassen *et al.*, 1960) concluded as follows:

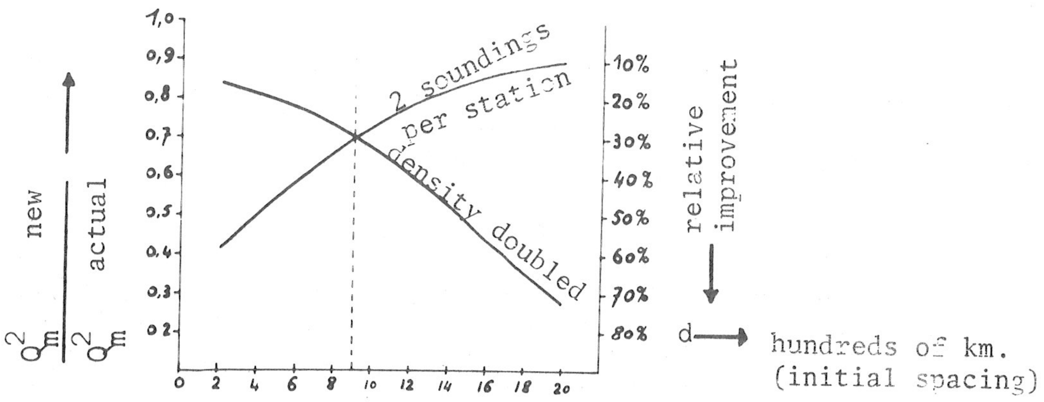


Fig. 5. Comparison of reduction in analysis error achieved by doubling station density and launching two simultaneous radiosondes from each station (Pône, 1960).

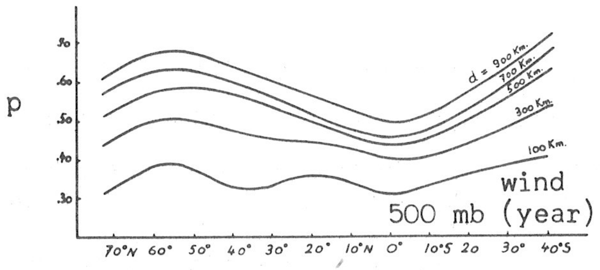


Fig. 6. Probability that σ_1 will not exceed K times σ_v

where σ_1 = 10 knots, when K = 1/2

or σ_1 = 5 knots, when K = 1/4

(Torrance, 1960)

"(a) In extratropical latitudes, a network of stations 500-600 km apart is adequate for numerical forecasting of isobaric surfaces if both height and wind are observed. For stations observing height only, the network should be closer (300-350 km between stations).

"(b) An increase in the frequency of observations from every twelve hours to every six hours could compensate for station density somewhat lower than the required density.

"(c) There is some indication that the required density is less over the sub-tropical anticyclones and more over the polar regions, but there is no reliable estimate of the permissible decrease of station density in the sub-tropics and of the required increase in higher latitudes."

The CSM Working Group on Networks (Bessemoulin et al., 1960), on the other hand, did not attempt to draw any firm conclusions concerning network requirements. However, in his contribution to the report of the Working Group, P. D. Thompson estimated from an assessment of average atmospheric conditions made from available information that, for no error growth, maximum spacing between stations should not exceed 400 miles in the temperate zone, about 700 miles in the polar regions and about 1000 miles in the tropics. These estimates do not take into consideration extreme conditions such as occur in the belt of strong westerlies.

It will be noted that Thompson's estimate concerning the relative density of stations required in temperate and polar latitudes is at variance with the conclusions of the CAe Working Group mentioned above.*

2.4. Minimum Station Spacing - Perhaps as important as the concept of maximum spacing (minimum density) of stations is that of minimum spacing (maximum density) which is intimately related to the efficiency (E_s) of a network, defined as

$$E_s = 1 - \frac{\sigma_i}{\sigma_v} = 1 - \lambda \tag{10}$$

where $\sigma_i = (\overline{\epsilon_i^2})^{1/2}$ is the standard deviation of the error of observations (ϵ_i) and σ_v is the standard variation, in time or space, of a meteorological parameter z. σ_v decreases with decreasing distance between stations and with increasing frequency of observations from a given station.

The minimum interval between stations is evidently one below which the error of observations becomes too large in comparison with the variability of the observed elements. The probability (p) that the instrumental errors do not exceed a certain fraction k of σ_v is given by the following relation proposed by Bessemoulin (1960):

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*Dobryshman (1964) similarly concludes that the required station density over polar regions is less than that over temperate latitudes since, in the polar troposphere, large horizontal gradients seldom occur.

$$p = 1 - \frac{2}{\pi}\tan^{-1}\left(\frac{1}{k}\cdot\frac{\sigma_i}{\sigma_v}\right) = 1 - \frac{2}{\pi}\tan^{-1}\left(\frac{\lambda}{k}\right) \qquad (11)$$

From a study of the available statistics, Torrance (1960) found that $_v$ increases slowly from the equator to 25° latitude, increases rapidly from 25° to 50°, and then decreases slowly between 50° and 90° latitude. These changes are reflected in Figs. 6 and 7, which give the probability that $_i$ does not exceed certain fractions of $_v$ under different conditions or different latitude belts. While these figures do not provide unique criteria for minimum station spacing, they clearly confirm that a higher network density is required for the middle latitudes than for the tropical and polar regions.

2.5. Efficient Utilization of Networks - In a remarkable compilation which draws on the considerable work done in USSR on the fundamentals of network design and which takes cognizance of studies made outside the USSR, Gandin (1963) stresses that the problem of determining network requirements cannot be divorced from the techniques of objective analysis since the same methods and the same information on the statistical structure of the atmosphere which are conducive to a good objective analysis can be brought to bear on the rational design of networks. Furthermore, the same network can produce different initial state errors with different analysis schemes.

2.5.1. Optimum Interpolation - Gandin discusses the advantages of optimal interpolation schemes such as the one utilized by Pone, which was discussed earlier. The advantage of such schemes lies, primarily, not so much in the fact that the schemes tend to minimize the mean error of interpolation as in the fact that they take into account the effects of observational errors.

For many meteorological elements (wind, humidity, etc.) the random errors are only slightly less than the variability of the element. Failure to take these into account may lead to the choice of inaccurate weighting factors and, hence, to an unadjusted increase in the interpolation error. By using an optimal interpolation scheme it is possible to determine the values of the given element everywhere in the region of interest, including the points of observation, with an accuracy exceeding that of the observations themselves. The prima facie importance of this technique to the accuracy of numerical forecasts should be more thoroughly investigated.

Mashkovitch (1964) has used optimal interpolation in an attempt to determine the distribution of the relative error of interpolation - defined as the ratio of the mean square error of interpolation to the mean square deviation of a meteorological element from its climatic value - corresponding to the present aerological network. His results, given in Fig. 8, indicate that this method allows a highly accurate analysis of the geopotential fields over Europe, America, and most of Asia. However, the relative errors are large over the Pacific and Indian Oceans and over the equatorial Atlantic.

2.5.2. Dynamic Analysis - The possibility of decreasing initial condition errors by special analysis techniques was also discussed by Thompson (1961) in connection with networks characterized by large areas where there are little or no data surrounded by regions where both the frequency and density

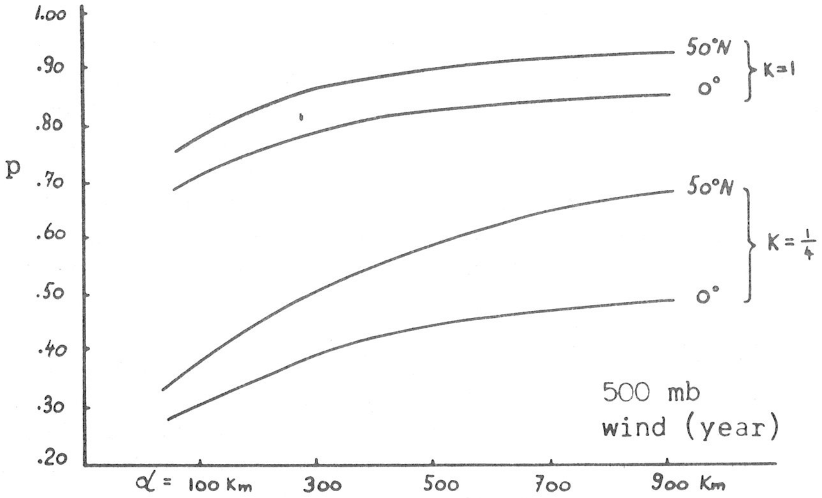

Fig. 7. Probability that σ_i will not exceed K times σ_v σ_i = 5 knots (Torrance 1960).

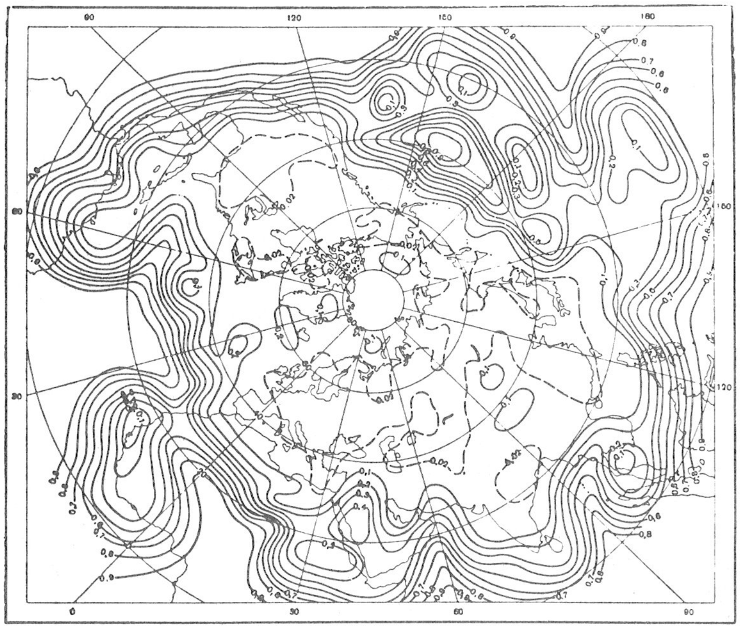

Fig. 8. Distribution of the relative error of optimal interpolation corresponding to the present aerological network (Mashkovitch, 1964).

of observations are fairly high. On the assumption that vorticity is nearly conserved along forward trajectories, Thompson proposed a method which combines numerical analysis and prediction procedures by which one can generate a series of increasingly exact analyses in the data "hole." The method was tested experimentally by Richardson (1961) and gave improved results much as predicted by theory.

3. EXPERIMENTAL STUDIES

A convenient though, perhaps, less fundamental approach to the problem of determining network requirements consists of comparing the results of numerical integrations of forecasting models made from initial data of varying densities. A numerical prediction is made from a reconstructed initial state, in which the data are very dense, and compared with another made from an initial state independently reconstructed with a fraction of these data.

This approach has the disadvantage of combining, in one collection of numerical values, errors inherent in the forecasting model used as well as errors which are a function of the meteorological situation considered. However, if a sufficient number of different situations are studied and the forecasts are made with different models, the method is capable of isolating the effect of different network densities and configurations on errors in forecasts produced by each model. The great advantage of this approach is that it circumvents the difficulties experienced in taking full advantage of the theoretical approach owing to the lack of sufficient statistical information on the structure and variability of the atmosphere.

Bristor (1958) was among the first to make use of this experimental approach by comparing numerical forecasts made by a barotropic model from four different analyses of two separate synoptic situations. He defined a reference atmosphere for each situation by interpolating, from a routine 500 mb operational analysis, values at 1977 grid points and postulated that these grid-point values represented the true heights of the 500 mb surface at the points in question. He then attempted to reproduce these "true" grid-point values. Fig. 9 shows the reference 1977-point grid (A) and the successively reduced array (B, C, D). The coverage represented by 1B closely approximates that over most of Canada, that represented by 1C approximates Atlantic coverage without reconnaissance data and 1D approximates Pacific coverage without reconnaissance data.

To simulate errors of observation, random normally disturbed errors were added both to the heights and to the geostrophic wind computed at each grid point. Fig. 10, in which the percentage of area with wind errors exceeding 20 kts. is plotted against the length of forecast period, is typical of the results obtained by Bristor. It shows that the area of large errors triples as the coverage changes from grid array B to D in Fig. 9.

Essentially the same approach was followed by Jess (1960) using Swedish Air Force routine numerical predictions to verify experimental forecasts from observations 600, 900, and 1200 km apart. An interesting feature of results obtained by Jess is illustrated in Fig. 11, which shows that when random

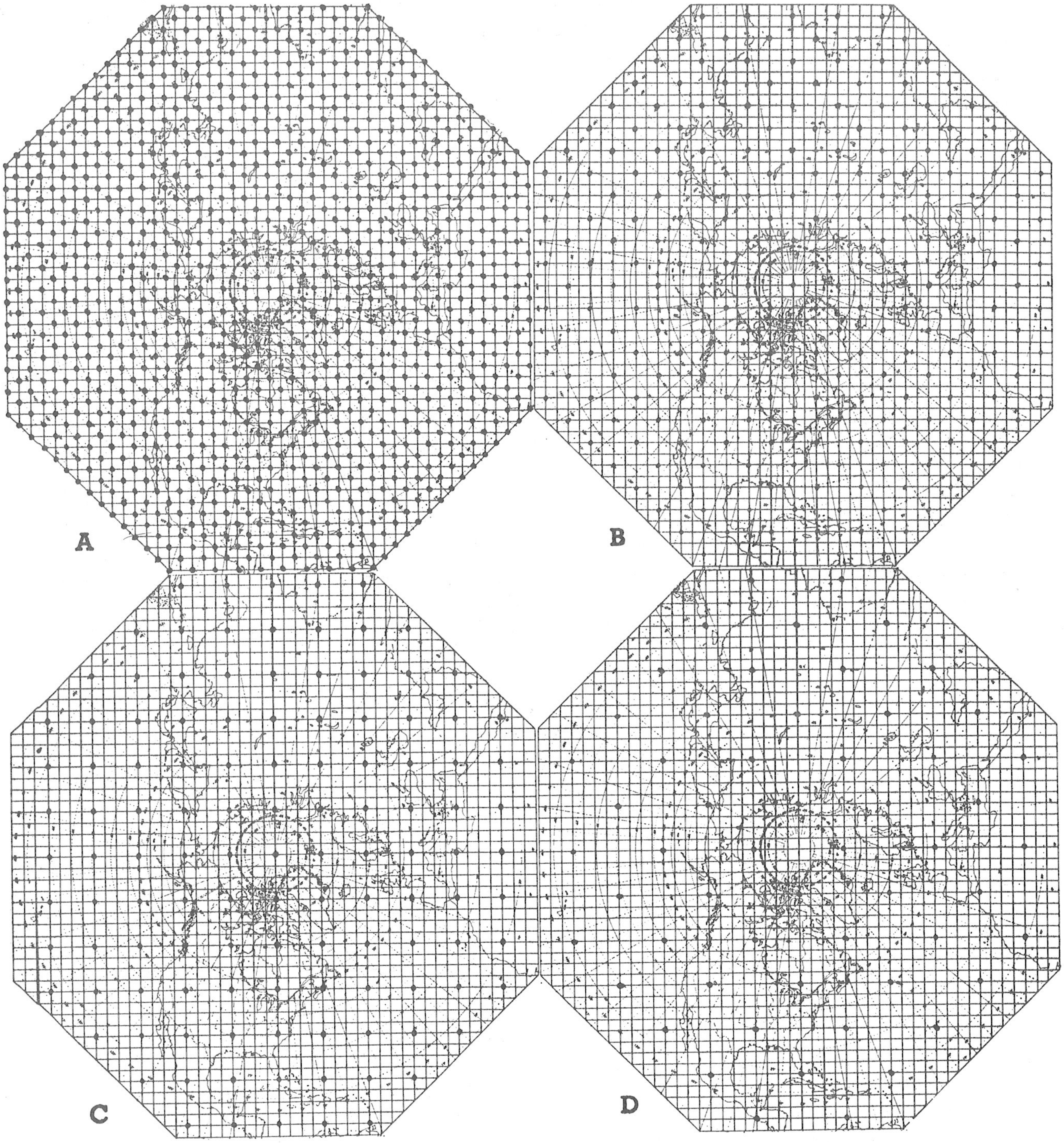

Fig. 9. Data coverage, shown by dots, for the octagonal grid of 1977 points currently used in the JNWP Unit (A) and successively reduced arrays (B, C, D) (Bristor, 1958).

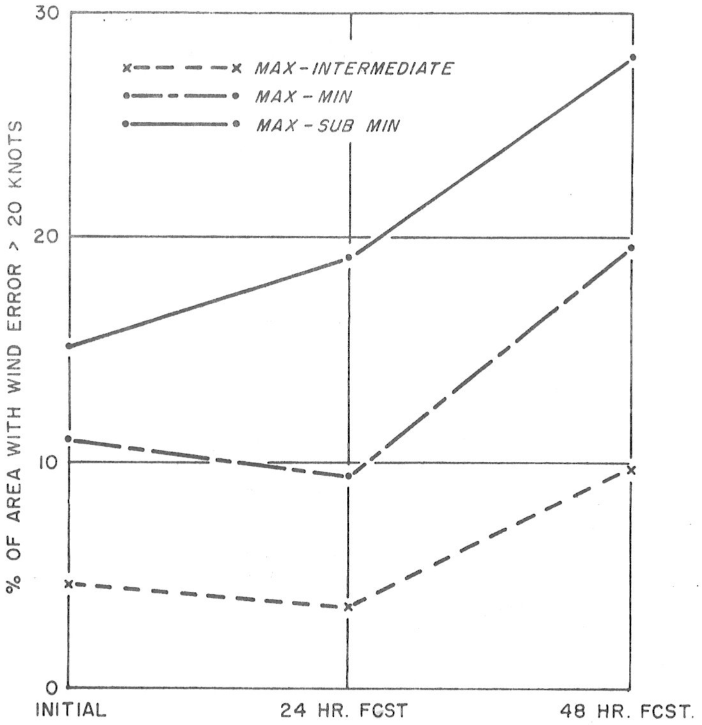

Fig. 10. Wind errors versus forecast range, Case 1, subjective analysis. Max = grid array A in figure 9, Intermdt = grid array B, Min = grid array C, and Sub-Min = grid array D (Bristor, 1958).

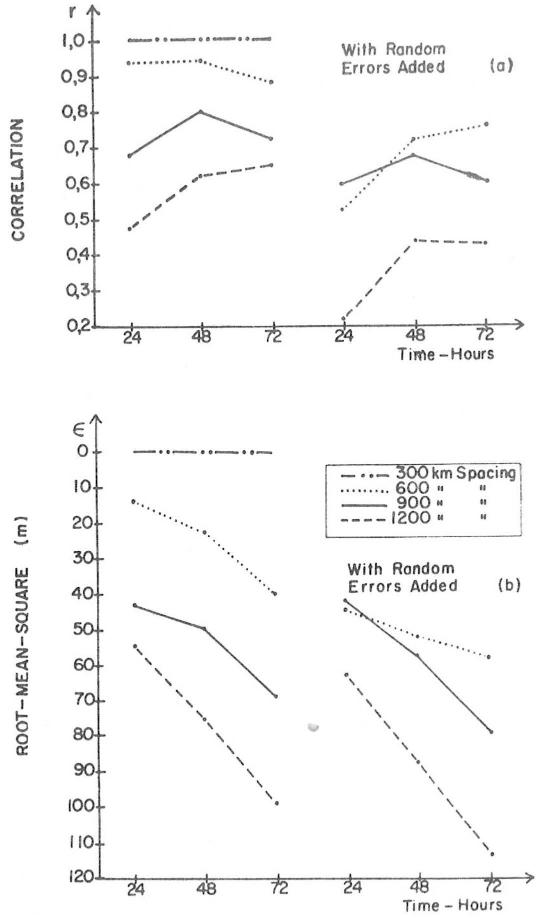

Fig. 11. Height Change Comparison. (a) Correlation between forecast and verifying changes (with time) of pressure-heights. (b) Root-mean-square of the difference between verifying and forecast height changes, (Jess, 1959).

errors are added to the data, the forecasts do not improve very greatly when the spacing between stations is reduced from 900 to 600 km. This result is compatible with that discussed earlier concerning relevance of observational errors in determining maximum station density.

Still another series of experiments was performed by an ad hoc panel appointed by the Chief of the Weather Bureau with the objective of determining the relationships between forecasting accuracy over the contiguous United States and the observational networks in sparse areas. The results of the experiments, described in Technical Planning Study No. 1 (1960), indicated that forecasting accuracy at 500 mb would not be greatly improved by a network having a spacing of less than 600-800 miles (Atlantic coverage) but that a network of lesser density will result in a sharp increase in forecast error (Fig. 12).

4. CONCLUSION

It is clear that considerable thought and effort have been devoted in the past decade to the problem of determining data requirements for numerical weather prediction. The fundamental theoretical studies have not only defined the dimensions of the problem but have also resulted in the elaboration of detailed techniques by which the required criteria could be derived. Yet, to date, results from such theoretical studies are in the nature of gross estimates which fall short of the considerable refinements of which the approach is potentially capable.

The reason is that the proper answers depend on a detailed knowledge of the structure and variability of the atmosphere, notably in the form of variances and autocorrelation functions which are not available except to a very limited extent. Gandin (1963) has discussed in detail the manner in which these statistics can be generated and the caution which must be exercised to insure reliable results. The impressive work of Buell (1962) also has a direct bearing on the problem and his results, or an extension thereof, may well provide a good foundation for future attempts to determine data requirements on the basis of the structure and variability of the atmosphere.

Similarly, the experimental studies, while shedding some interesting light, have for the most part been designed with limited objectives. They do not exhaust the possibilities of this approach.

The net result is that, although several estimates of aerological data requirements have been made, there is a large disparity in these estimates, as can be seen from Table 1. This disparity is probably responsible for the fact that the official criteria on network requirements, adopted by WMO and published as part of Technical Regulations of this organization, are precisely those recommended twenty years ago by the Paris Session of the Commission for Synoptic Weather Information.

Clearly, more work needs to be done in this area.

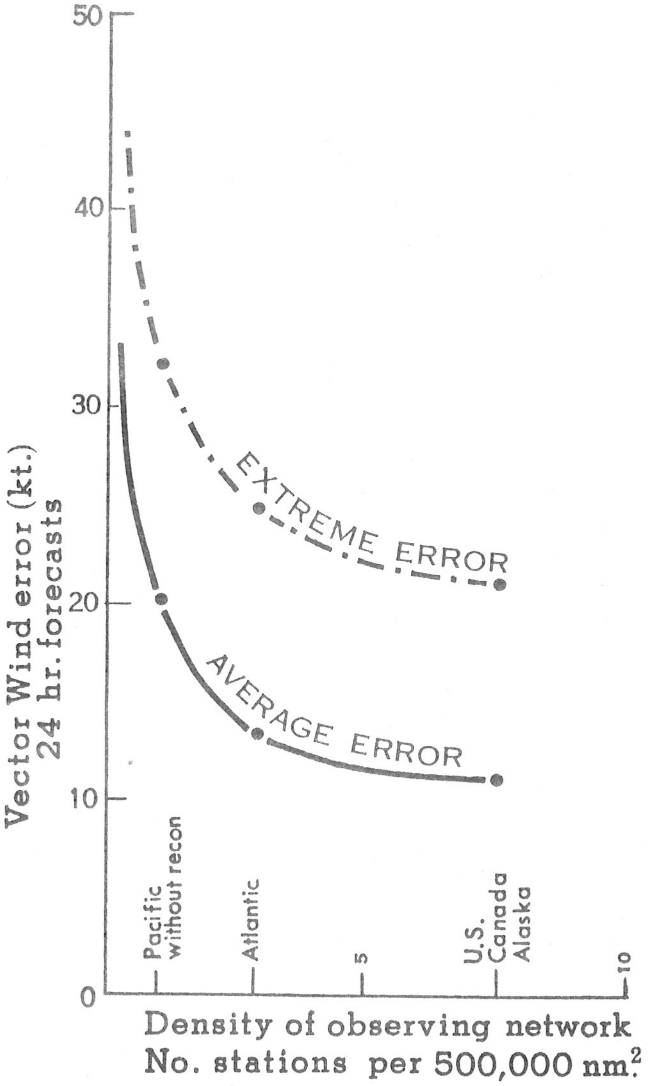

Fig. 12. Variation with network density of vector wind error in 24-hr. forecasts (Panel on Observations Over Sparse Data Regions, 1963).

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