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An Experiment in the Use of the Balance Equation in the Tropics

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AN EXPERIMENT IN THE USE OF THE BALANCE EQUATION IN THE TROPICS

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ABSTRACT

The balance equation is used to generate the geopotential height fields from the wind fields at 850, 500 and 200 millibars during the development of hurricane Carla (1961).

The results show that, although the generated heights are consistently lower than the reported heights, the differences are within the range of instrumental errors, even at 200 millibars where strong divergence occurs in connection with outflow from the storm.

Comparison of the 200-mb contours generated by the balance equation with those obtained by including the divergence term shows that the latter heights exceed the former by up to 80 ft. The maximum differences, predictably, are found where the highest values of the divergence occur.

I INTRODUCTION

The comparative usefulness of pressure and wind observations in the tropics has been the subject of much dispute and controversy. In contrast with extra-tropical regions where synoptic-scale disturbances can be delineated by several closed isobars drawn at an interval which exceeds the probable error of observations, synoptic-scale weather systems in the tropics often appear as relatively small perturbations whose magnitude is comparable with the probable observational errors.

At the surface, local influences may introduce variations which completely mask the features which are intrinsic to the synoptic system itself. In the upper air, while such local noise is greatly reduced, pressure observations made by radiosondes depend on an upward integration from measured surface values, and the errors tend to be both systematic and accumulative. In higher latitudes this defect is counterbalanced by the tendency for pressure-height gradients to increase in the upper troposphere. This redeeming feature is not particularly noticeable in the tropics.[4]

Of all directly measurable meteorological elements, the upper winds are probably most representative of synoptic-scale disturbances. Even near the surface where local and convective influences can be large, conditions representative of these systems may be approximated by averaging wind observations over a suitably long period. It would, of course, be helpful if accurate delineation of the synoptic pressure field, uncontaminated by excessive observational errors and local and smaller scale influences, could be derived from the wind field. The traditional and most simple relationships used for this purpose are the geostrophic and gradient-wind approximations which have been found satisfactory in higher latitudes. However, opinions are divided concerning the validity of these approximations in the tropics, especially the geostrophic-wind approximation.[4,5]

With the advent and increasing availability of high speed electronic computers, it has become feasible to test the suitability, in the tropics, of more complex relations between wind and pressure-height fields. The purpose of this article is to present the results of a simple experiment designed to test the accuracy with which the balance equation can be used to generate geopotential height fields from wind fields in the tropics.

II THEORETICAL BACKGROUND

It will be recalled that the geostrophic approximation is a filtering device which excludes from the general hydrodynamical equation solutions corresponding to the troublesome gravity waves which are extraneous to the weather producing processes.[1] But it turns out that this approximation, though sufficient, is not necessary for filtering purposes. Let us consider the horizontal equations of non-viscous motion

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \omega\frac{\partial u}{\partial p} = -\frac{\partial \phi}{\partial x} + fv \qquad (1)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \omega\frac{\partial v}{\partial p} = -\frac{\partial \phi}{\partial y} - fu$$

where t denotes time, Φ is the geopotential (gZ), f the Coriolis parameter, p the pressure and $\omega = \frac{dp}{dt}$. A horizontal divergence equation may be obtained by differentiating the above equations with respect to X and y and adding. The result after combining terms is:

$$\frac{dD}{dt} - 2J(u,v) + D^2 + \beta u - f\zeta + \left(\frac{\partial \omega}{\partial x}\right)\frac{\partial u}{\partial p} + \left(\frac{\partial \omega}{\partial y}\right)\frac{\partial v}{\partial p} + \nabla^2\phi = \phi \qquad (2)$$

In the above equations

$$D = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \quad \text{(divergence)}$$

$$J(u,v) = \frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial v}{\partial x}\frac{\partial u}{\partial y}$$

$$\beta = \frac{\partial f}{\partial y}$$

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2}$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + \omega\frac{\partial}{\partial p}$$

As shown by Thompson[10], the necessary and sufficient condition for filtering equation (2) is:

$$\frac{dD}{dt} = 0 \qquad (3)$$

If the motion is assumed isobaric

$$\omega = \frac{\partial w}{\partial x} = \frac{\partial w}{\partial y} = 0 \qquad (4)$$

and equation (2) reduces to:

$$\nabla^2 \phi = 2J(u,v) - D^2 - u\beta + f\zeta \qquad (5)$$

Assuming the motion to be non-divergent, $D^2 = 0$ and equation (5) reduces to the familiar balance equation which may be expressed in terms of a stream function (Ψ)

$$\nabla^2 \phi = f\nabla^2 \Psi + 2\left[\frac{\partial^2 \Psi}{\partial x^2}\frac{\partial^2 \Psi}{\partial y^2} - \left(\frac{\partial^2 \Psi}{\partial x \partial y}\right)^2\right] + \nabla\Psi \cdot \nabla f \qquad (6)$$

where

$$\frac{\partial \Psi}{\partial y} = u$$

$$\frac{\partial \Psi}{\partial x} = -v$$

$$\zeta = \nabla^2 \Psi$$

It will be noted that the validity of the above equation rests on the premise that the motion is isobaric and non-divergent. Charney[2] has shown, by the process of scale analysis, that large-scale atmospheric motion in the tropics is indeed quasi non-divergent in the absence of condensation. Under such conditions one can surmise a-priori that equation (6) would be satisfied to an acceptable degree of approximation. But it remains to be demonstrated that the relation also holds under certain other conditions marked by copious condensation and abnormally large divergence such as occur in hurricanes.

For this reason, the time chosen for the present experiment covered the 24-hour period from 0000Z, September 4 to 0000Z, September 5, 1961 at the time when Carla was rapidly reaching hurricane intensity. Fortunately, this occurred in an area with good data coverage, thus allowing a reliable analysis of the wind field which provided the main data source for the experiment. Figure 1 shows the successive positions of the incipient hurricane during the period under study. In this figure the network of upper air observing stations is denoted by the triangles; the dots represent the grid used in the computations.

III THE WIND FIELD

Detailed wind analysis was made at 850, 500 and 200 mb for the period under study using data from the radiosonde stations, supplemented by data from a reconnaissance flight at 500 mb on September 4. Streamlines were carefully drawn from the initial isogon analysis, care being taken to maintain continuity both in time and space. Allowance was made, in the 850 mb analysis, for the mountains of Central America since it was found that these

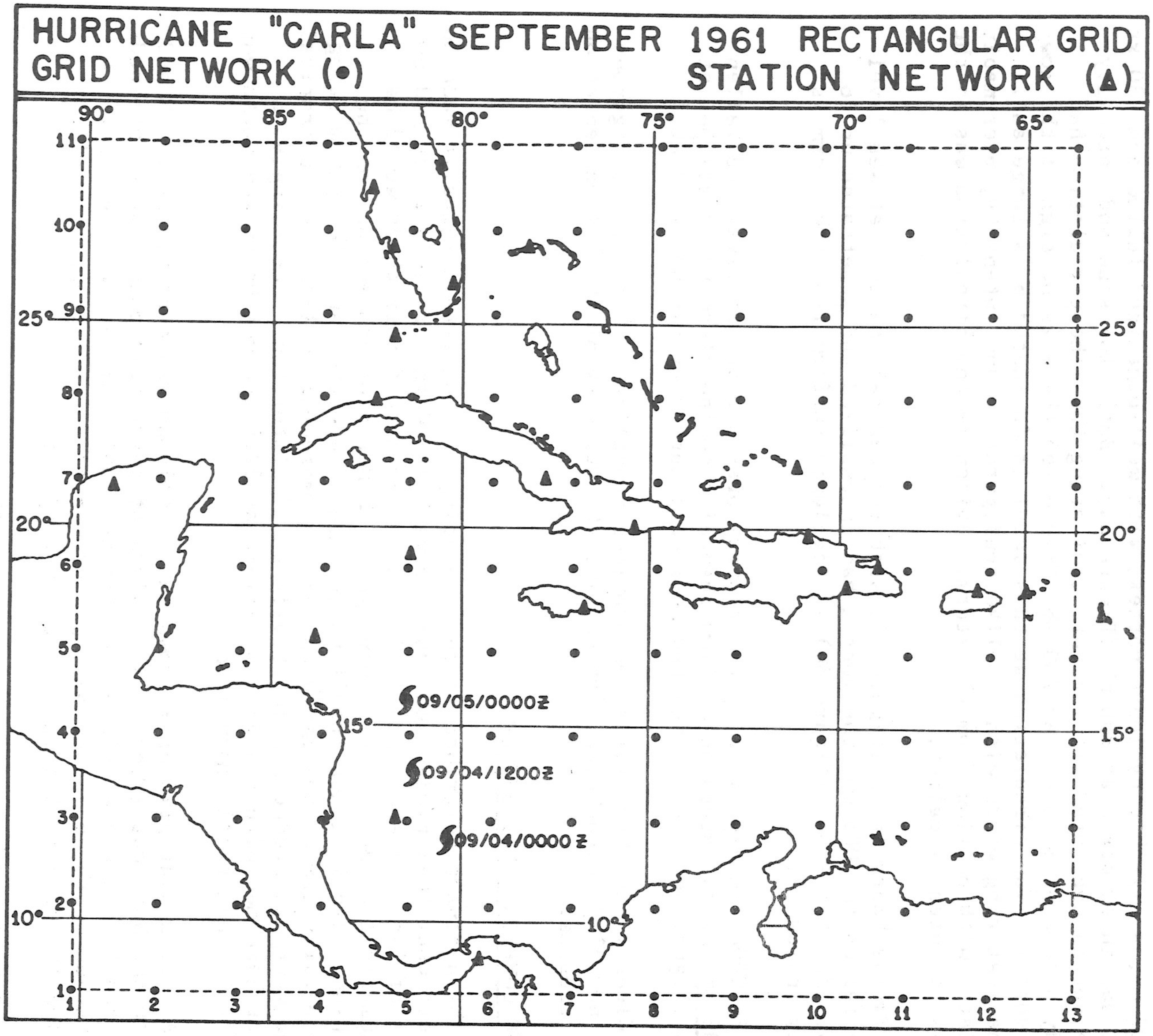

Figure 1. The grid used in the computations. The upper-air observing stations are denoted by triangles, and the successive positions of the incipient storm during the period under study are indicated by the hurricane symbol.

mountains significantly altered the streamline pattern which might otherwise be found in the region. The analyzed wind fields are shown in figures 2 - 4.

At 0000Z on September 4, with an estimated central sea-level pressure of 1007 mb, a sizeable cyclonic vortex, with well-organized inflow and 30 kt winds, was in evidence at 850 mb (Figure 2a). Both the size and intensity of the vortex decreased with height, indicating that the circulation had already become warm-core. At 200 mb (Figure 2b) there is no sign of the low-level vortex. The circulation at this level is dominated by a large anti-cyclone featuring winds with jet-like configuration and intensity, presumably associated with outflow from the storm. Evidently the circulation was in the process of intensification.

Twelve hours later (Figure 3), with the central pressure at sea level having fallen to 1005 mb, the vortex had intensified and penetrated up to 200 mb. At this level (Figure 3b) anticyclonic outflow from the storm is clearly indicated.

By 0000Z on September 5, the central pressure at sea level had fallen to 1003 mb and an extensive outflow channel from the storm had developed at 200 mb (Figure 4b). Carla reached hurricane intensity ten hours later.

Clearly the above situation provides conditions, which are as rigorous as any normally encountered, for testing the validity of the balance equation in the tropics.

IV THE COMPUTATIONAL SET-UP

The analyses described in the preceding section were originally made on a Lambert Conformal projection base map with standard parallels at 30 and 60 degrees of latitude and a scale of $1:10^6$. The use of this chart would have made numerical computations inconvenient. A Mercator Projection, on the other hand, readily lends itself to the use of a Cartesian grid system owing to the parallelism of the meridians and is, moreover, well suited to tropical regions where it produces a relatively small distortion. Accordingly, a Mercator base map with standard parallels at 22.5 degrees North and South was used for the numerical computations.

A rectangular grid of 11 rows and 13 columns was centered at 19° N and 77° W on the base map (Figure 1). Values at these grid points were obtained by reference to the analysis on the Lambert Conformal maps of figures 2 - 4. This procedure, though laborious, was preferred to the alternative of re-analyzing the wind fields on a Mercator Projection.

The distance between grid points is 120 nautical miles at 22.5° L. At other latitudes, the distance is given by the relation

$$d = \frac{120}{\sin 67.5 \csc \Psi} \qquad (8)$$

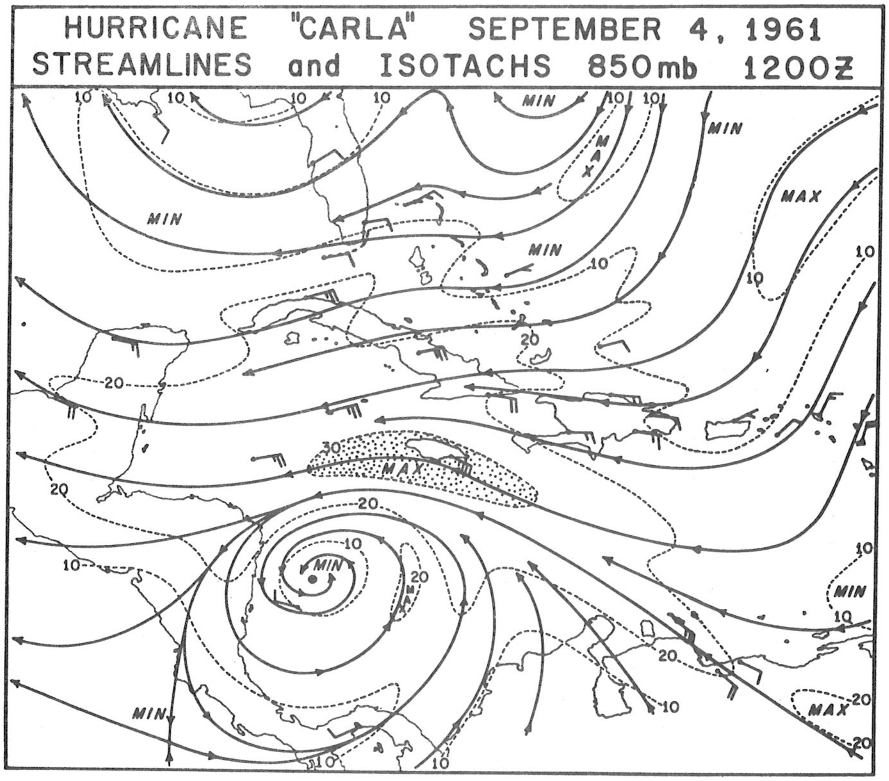

Figure 2a. 850-mb streamlines and isotachs (kts) at 0000Z, September 4, 1961.

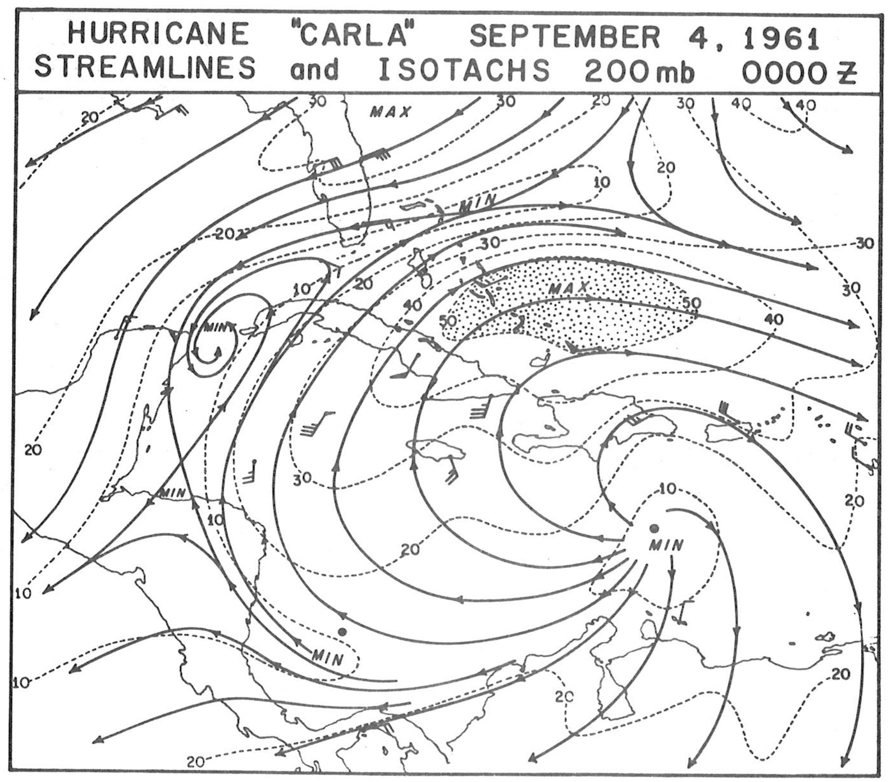

Figure 2b. 200-mb streamlines and isotachs (kts) at 0000Z, September 4, 1961.

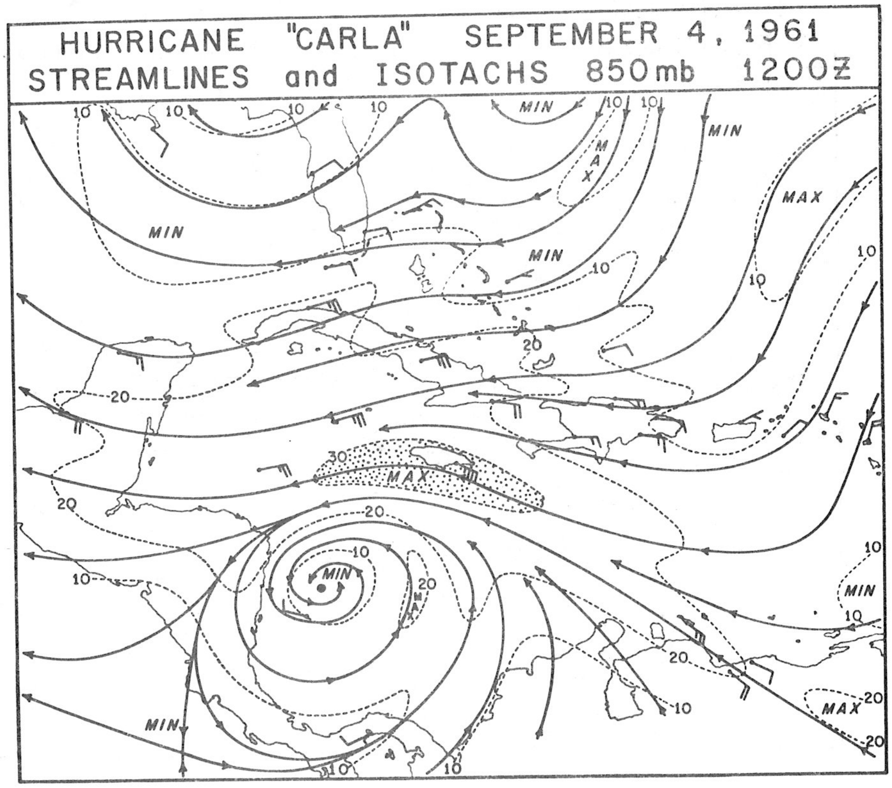

Figure 3a. 850-mb streamlines and isotachs (kts) at 1200Z, September 4, 1961.

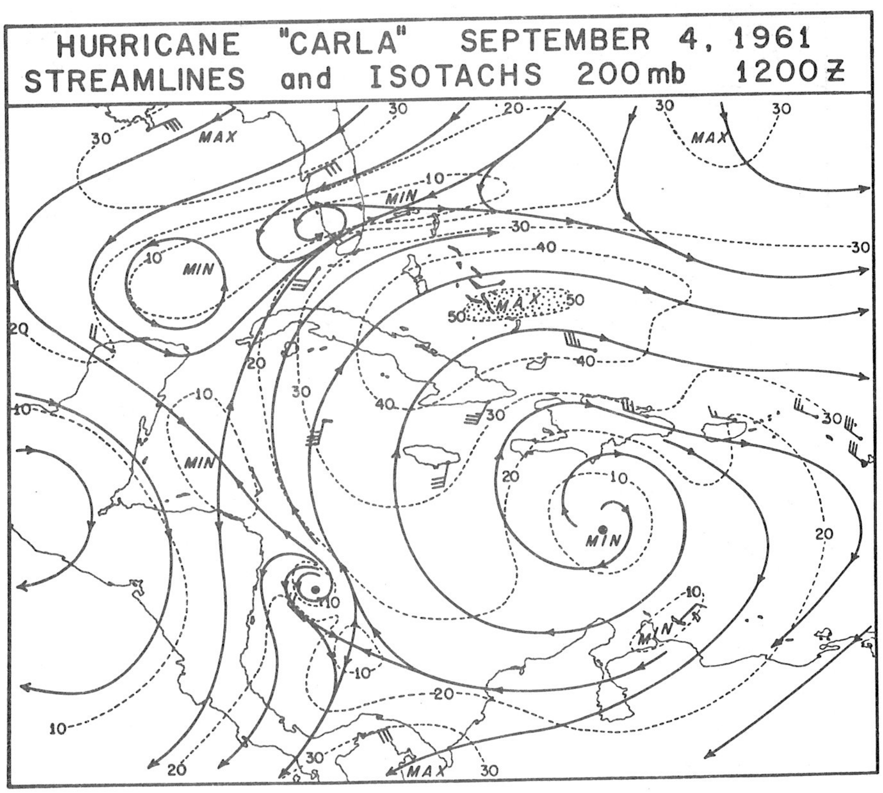

Figure 3b. 200-mb streamlines and isotachs (kts) at 1200Z, September 4, 1961.

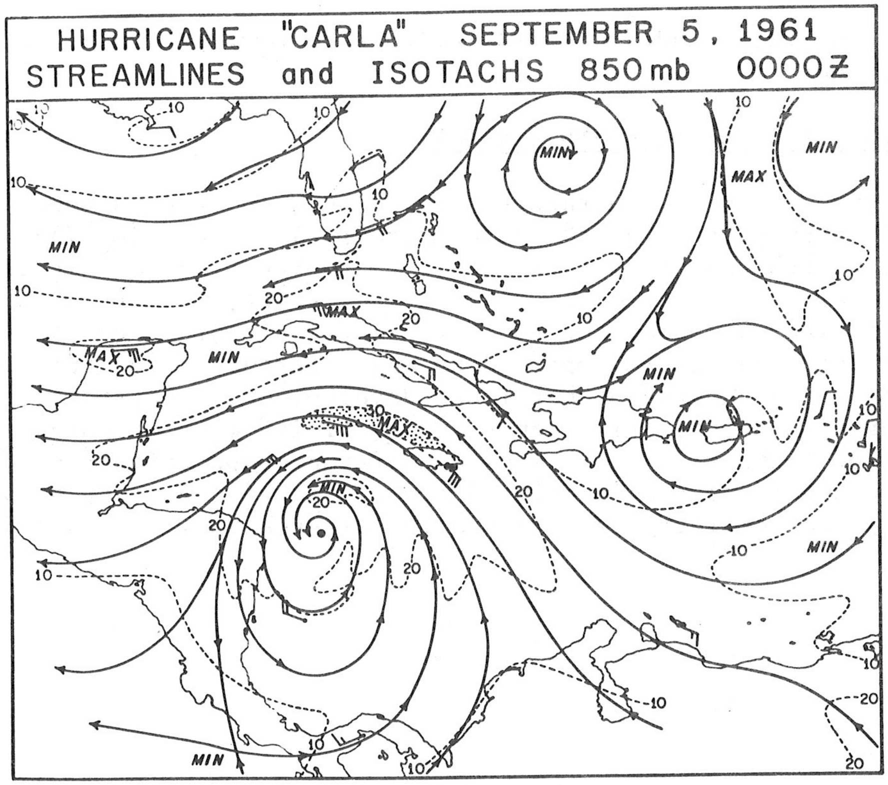

Figure 4a. 850-mb streamlines and isotachs (kts) at 0000Z, September 5, 1961.

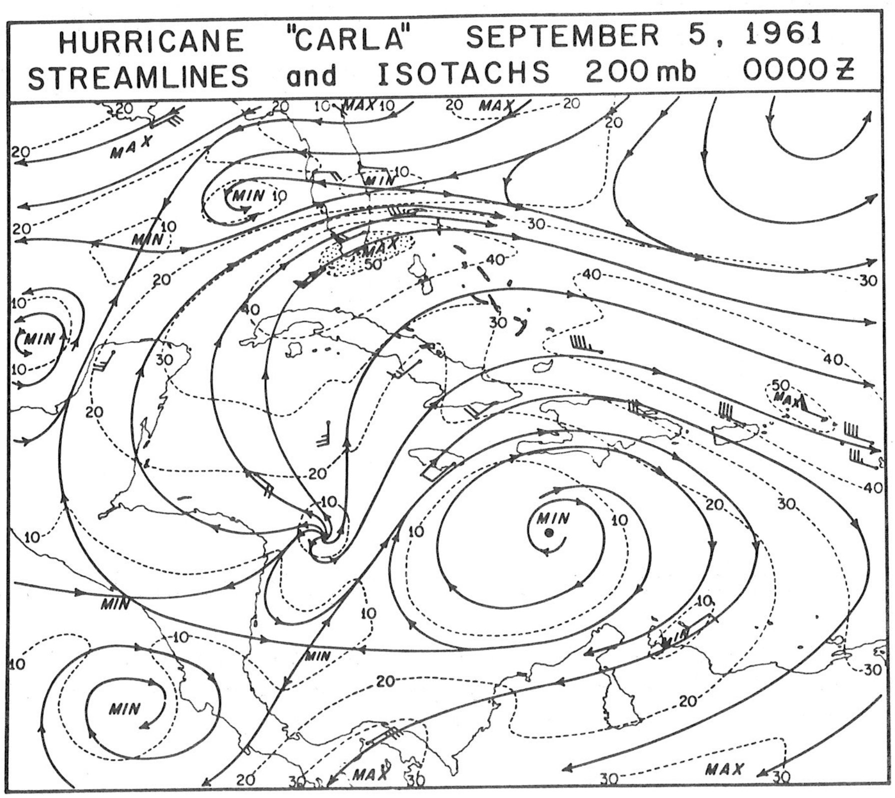

Figure 4b. 200-mb streamlines and isotachs (kts) at 1200Z, September 5, 1961.

where Ψ is the colatitude.[8] The variation of d over the entire grid is less than 10 per cent.

V THE FIELDS OF DIVERGENCE AND VORTICITY

As mentioned above, in theory, the validity of the balance equation rests on the assumption that the divergence term in equation (5) is negligible in comparison with the other terms, say, the vortcity term. It is therefore interesting to find out to what extent this assumption is true under the special conditions selected for the experiment, and also to what extent the validity of the equation is impaired by any departures from this assumption which obtain in the course of hurricane development. Therefore, at each of the interior grid points, divergence and vorticity were computed from the wind fields of figures 2 - 4 by finite-difference approximation. If the subscripts c and r refer respectively to column and row, then at any interior point (r, c) the vorticity ($\zeta_{r,c}$) and the divergence ($D_{r,c}$) are given by

$$\zeta_{r,c} = \left(\frac{\Delta v}{\Delta x}\right)_{r,c} - \left(\frac{\Delta w}{\Delta y}\right)_{r,c}$$

$$D_{r,c} = \left(\frac{\Delta u}{\Delta x}\right)_{r,c} + \left(\frac{\Delta v}{\Delta y}\right)_{r,c} \qquad (9)$$

where u and v are the zonal and meridional wind components, respectively, and x and y are distances in a Cartesian Coordinate system in which x is positive to the east and y positive to the north. The horizontal wind shears at each of the 104 internal grid points are approximated by the following:

$$\left(\frac{\Delta v}{\Delta x}\right)_{r,c} = (v_{r,c+1} - v_{r,c-1})/d_r$$

$$\left(\frac{\Delta v}{\Delta y}\right)_{r,c} = (v_{r-1,c} - v_{r+1,c})/d_r \qquad (10)$$

$$\left(\frac{\Delta u}{\Delta x}\right)_{r,c} = (u_{r,c+1} - u_{r,c-1})/d_r$$

$$\left(\frac{\Delta u}{\Delta y}\right)_{r,c} = (u_{r-1,c} - v_{r+1,c})/d_r$$

$$\overline{H}_{r,c} + \frac{4H_{r,c} + 2(H_{r,c+1} + H_{r,c-1} + H_{r+1,c} + H_{r-1,c}) + H_{r+1,c+1} + H_{r+1,c-1} + H_{r-1,c-1} + H_{r-1,c+1}}{}$$

To eliminate minor wiggles and thus bring out more clearly the major features in each field, the following 9-point smoothing procedure was adopted. If $\overline{H}_{r,c}$ denotes the smoothed value of a variable $H_{r,c}$ then

Since the divergence and vorticity were computed only at the interior grid points, an end-of table, non-centered difference modification of the above smoothing technique was applied near the boundaries. Figures 5 and 6 are representative of the computed vorticity and divergence fields.

Considering a possible error of 5 knots in the speed and 10° in the direction of the wind, the maximum error in the vorticity and divergence was computed to be of the order of 10^{-5} sec^{-1}. By this criterion, some doubt is cast on the accuracy of the computed fields of divergence at both the 850 and the 500 mb levels. Indeed at these levels, where values of the relative vorticity exceed 10^{-4} sec^{-1} near the center of the storm, the computations attest the validity of the assumptions on which the balance equation is based. While this result could have been expected at 500 mb, which lies in the neutral layer between the low-level convergence and the upper-level divergence, it is somewhat surprising that the same situation obtains as low as 850 mb. This would seem to indicate that the low-level inflow of the developing storm is concentrated in a shallow layer below this level.

At 200 mb the situation is quite different. Here the divergence values are comparable in magnitude with those of the relative vorticity, and the patterns reveal a well-defined axis of divergence to the north and east of the developing storm.

VI THE HEIGHT FIELDS

The finite-difference form of equation (6) used in the numerical calculation of the pressure heights is:

$$(\nabla^2 Z)_{r,c} = \frac{1}{g}\left\{2\left[\left(\frac{\Delta u}{\Delta x}\right)_{r,c}\left(\frac{\Delta v}{\Delta y}\right)_{r,c}-\left(\frac{\Delta u}{\Delta y}\right)_{r,c}\left(\frac{\Delta v}{\Delta x}\right)_{r,c}\right]-u_{r,c}\beta_r+f_r\zeta_{r,c}\right\} \quad (12)$$

where Z is the height of a pressure surface and g is gravity.

Equation (12) is a Poisson's equation in two dimensions which, with the addition of boundary heights, can be solved to give approximate values at each of the interior grid points. In the present case, Liebmann's Method of Relaxation was used to solve the equation. An over-relaxation coefficient of 0.4 was used[9] and the relaxation process was repeated until the largest residual in the height field was less than two geopotential feet - which was the tolerance specified for the experiment. Between 10 and 15 passes were normally required to converge to this value.

The height fields generated by the above process are shown in Figures 7-9. They show good general correspondence with the wind field, especially

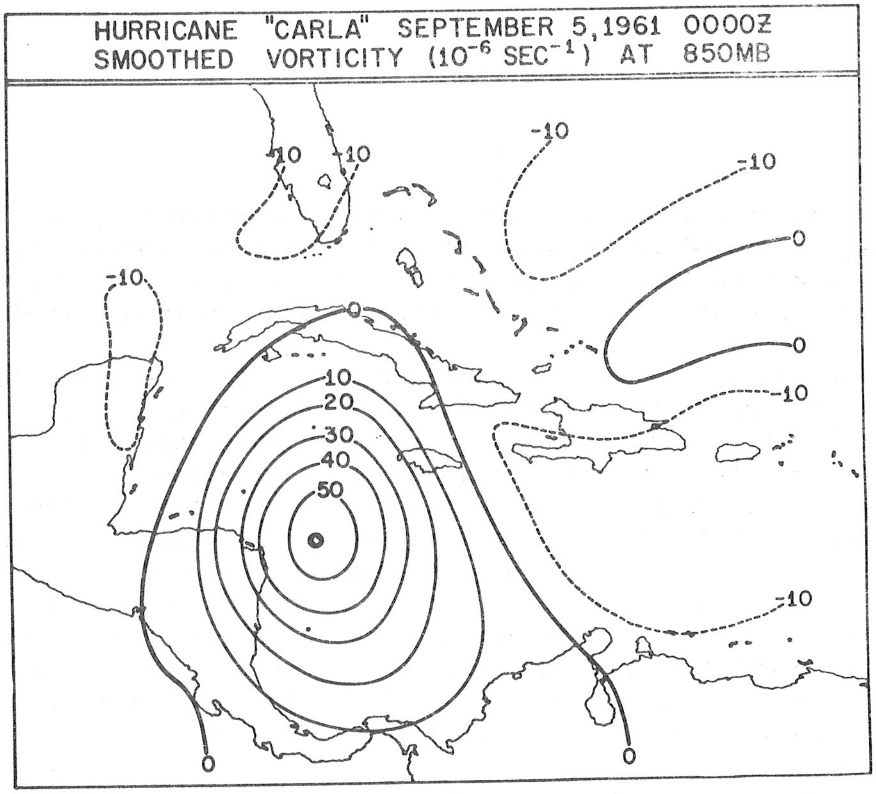

Figure 5a. Vorticity at 850 mb, September 5, 1961, 0000Z.

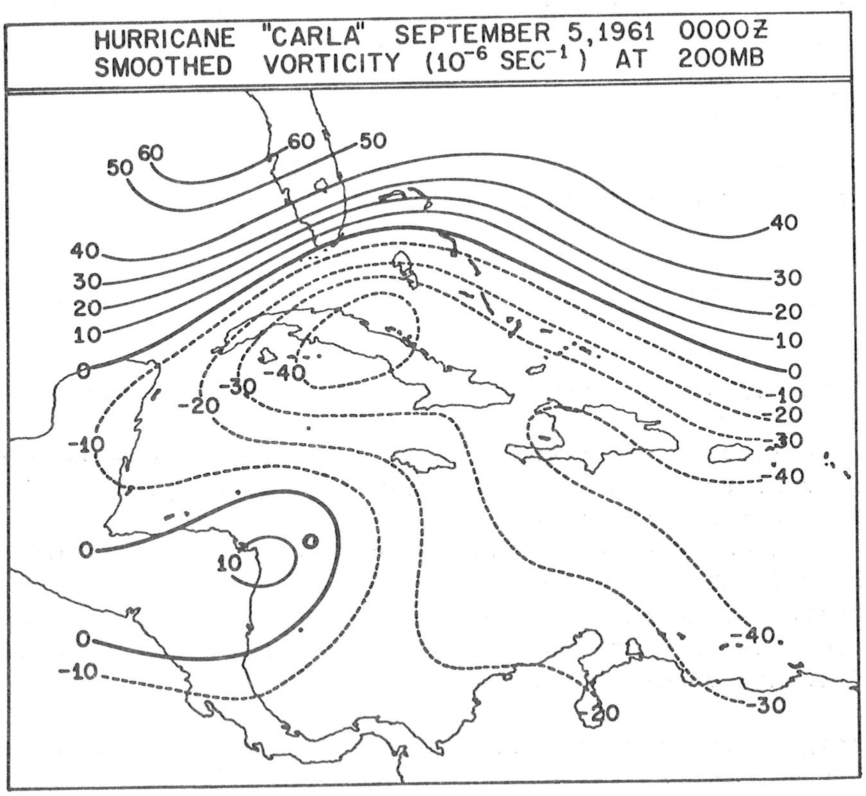

Figure 5b. Vorticity at 200 mb, September 5, 1961, 0000Z.

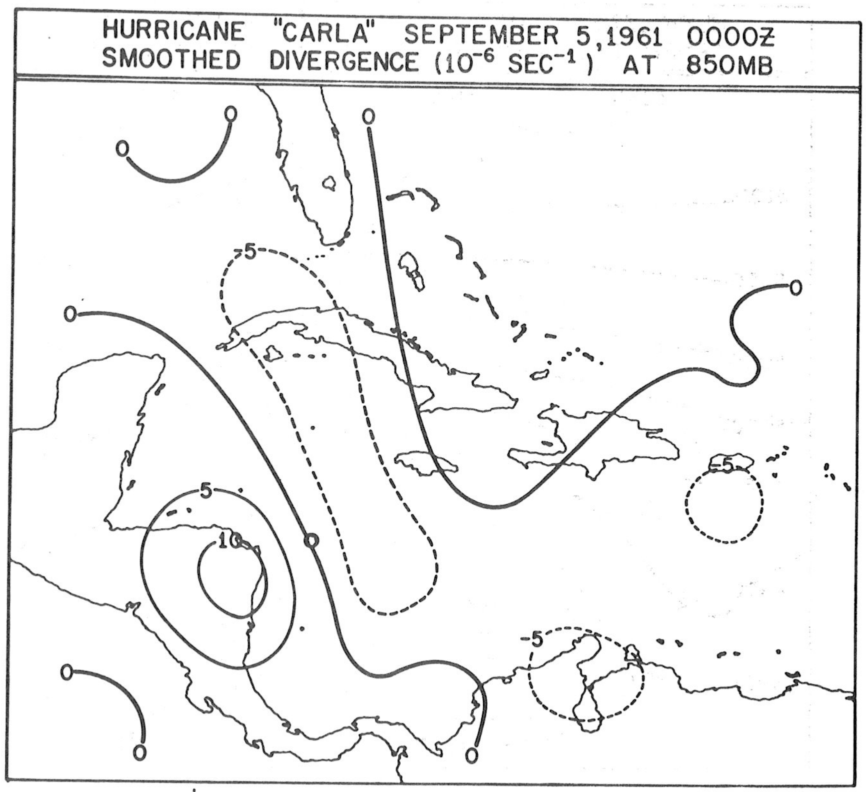

Figure 6a. Divergence at 850 mb, September 5, 1961, 0000Z.

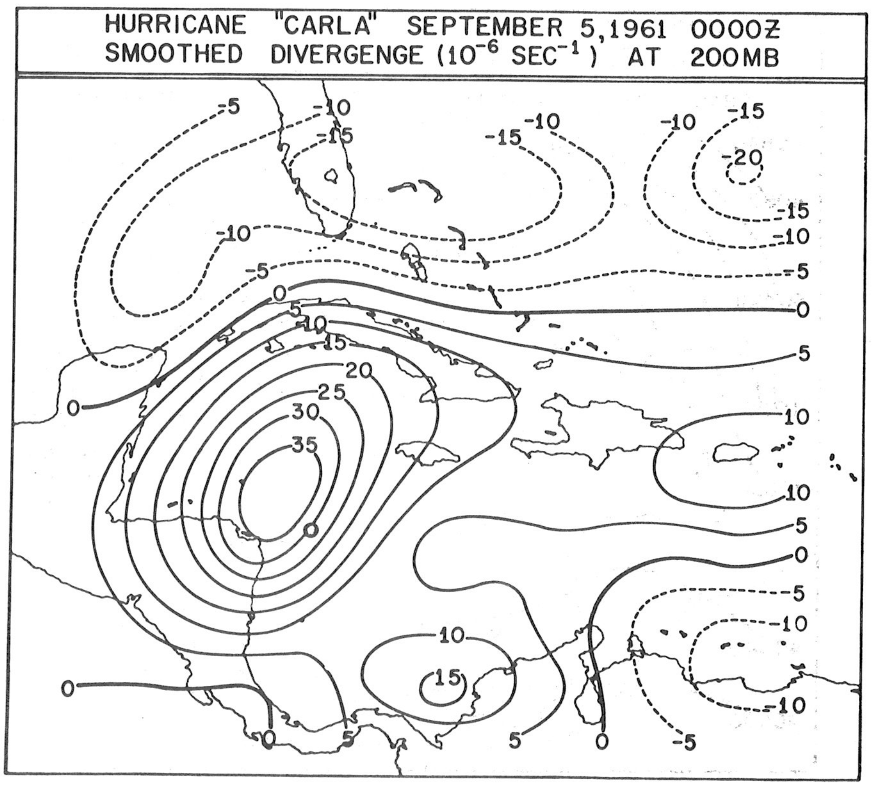

Figure 6b. Divergence at 200 mb, September 5, 1961, 0000Z.

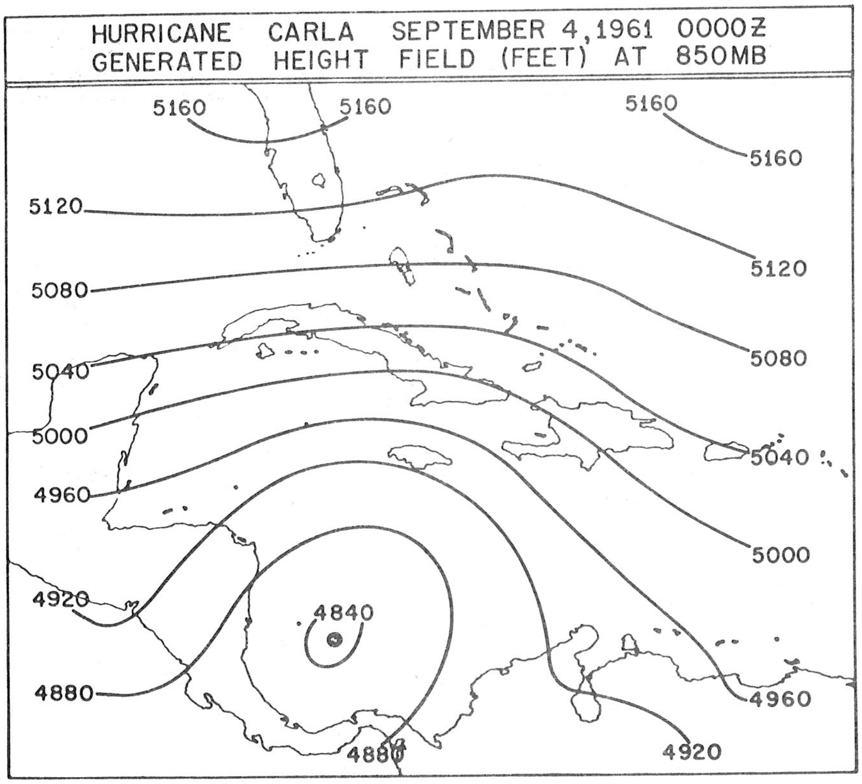

Figure 7a. Generated 850-mb contours, September 4, 1961, 0000Z.

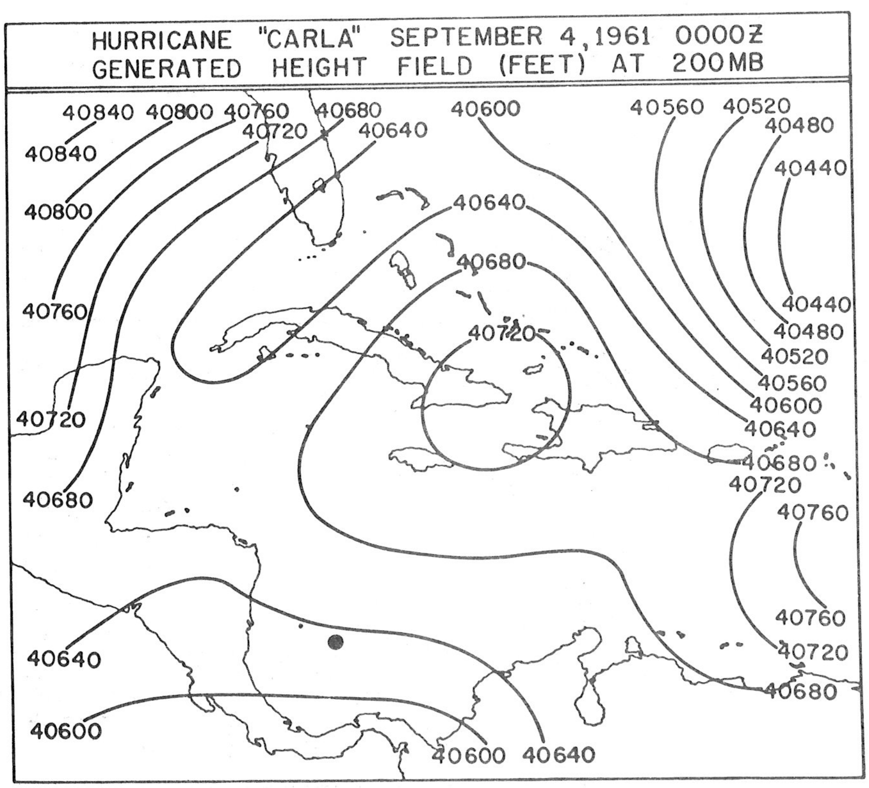

Figure 7b. Generated 200-mb contours, September 4, 1961, 0000Z.

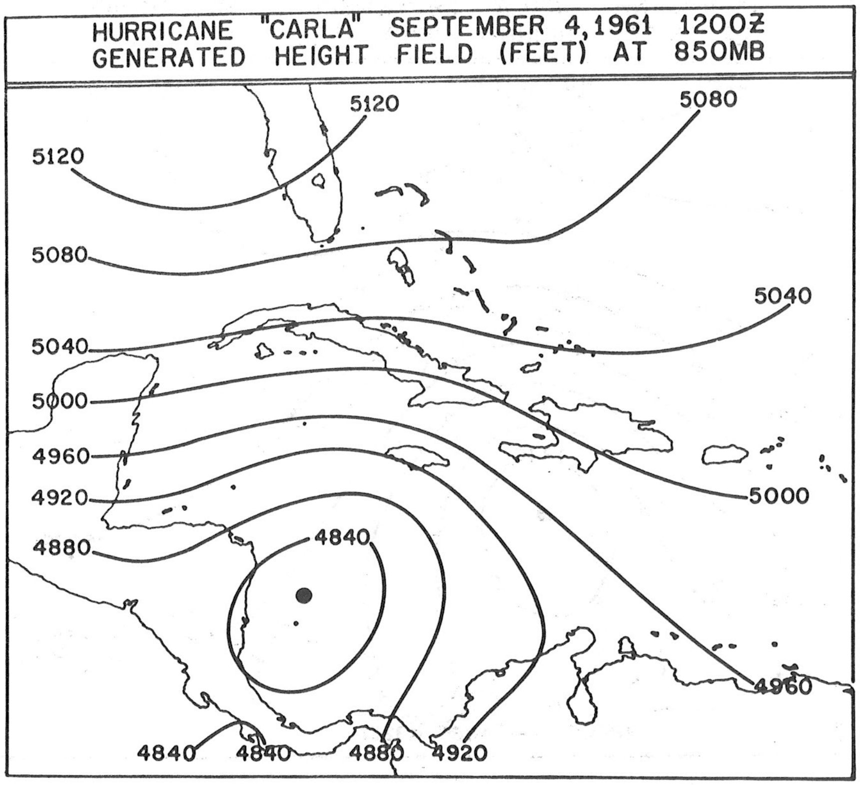

Figure 8a. Generated 850-mb contours, September 4, 1961, 1200Z.

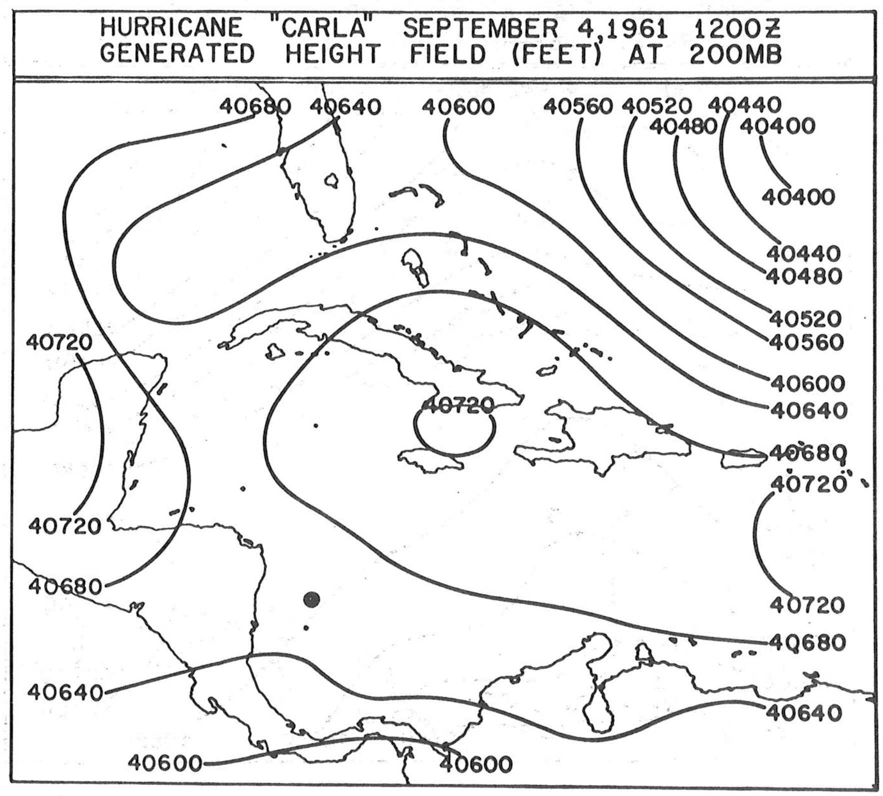

Figure 8b. Generated 200-mb contours, September 4, 1961, 1200Z.

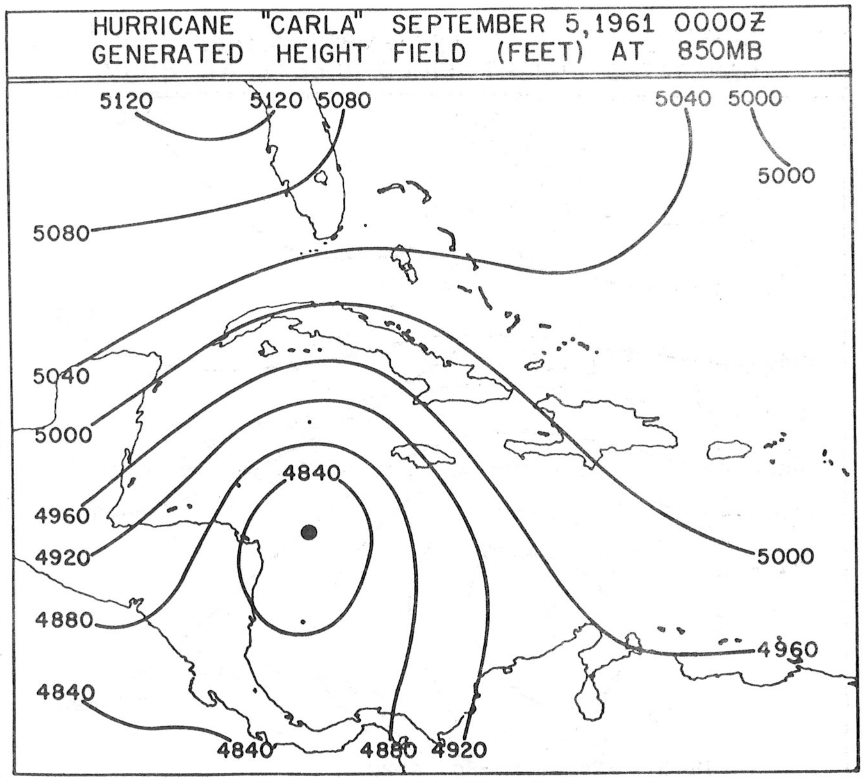

Figure 9a. Generated 850-mb contours, September 5, 1961, 0000Z.

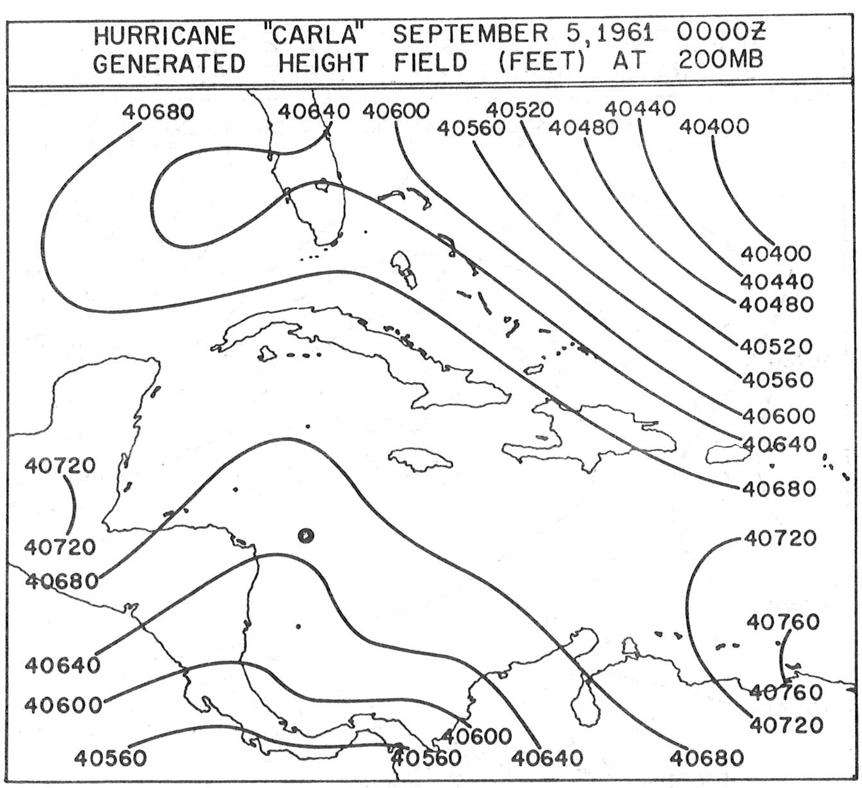

Figure 9b. Generated 200-mb contours, September 5, 1961, 0000Z.

at the lower levels. At 200 mb a trough in the height field is indicated in the northwest section where the streamline analysis shows a shear line. However, the representation of the cyclonic vortex at this level as a weak trough may be under-done. The grid interval of approximately 120 n.m. is probably too large to delineate this relatively small feature.

Table 1 gives some statistics on the difference of contour heights computed from radiosonde observations with heights at the same localities interpolated from the grid-point values generated by the balance equation. Both the mean and median differences are small. Thus if all stations within the grid are considered, the mean difference at 850 mb for the three maps periods ranges from 17 to 38 feet, while the median difference ranges from 13 to 22 feet. The maximum difference at this level was found to be 65 feet. If only interior stations are considered, the corresponding differences are 5 - 19 ft., 8 - 15 ft. and 36 ft. respectively. In all cases the balance equation underestimates the heights.

The differences increase upward and are at their highest at 200 mb. But even at this level, the mean difference ranges from 47 to 68 feet. This is not too bad considering that recent comparisons between simultaneous observations made by Weather Bureau and military radiosondes showed mean differences of 22 ft. and 33 ft., respectively, for daytime and night-time ascents (Hodge and Harmantas, 1965). Of course the disparity between results from radiosondes from different countries would be much greater, as can be seen from Table 2 which gives the results of comparisons of different types of radiosondes, made at Payerne, Switzerland, under the auspices of WMO.[6] The Payerne comparisons seem to indicate that the RMS random error in measurements of heights for all types of instruments is about 65 ft. at 500 mb, 100 ft. at 300 mb and 130 ft. at 200 mb.[7]

It will be noted from Table 1 that the reported heights are consistently higher than the generated heights. This is partly due to the fact, discussed by Hodges and Harmantas,[3] that flight similitude factory tests of the radiosondes showed a temperature about 0.4° too high. This would over-estimate the height of the 200 mb surface by about 62 feet. However, the differences in Table 1 are also partly attributable to the omission of the divergence term in the balance equation. It was therefore thought of interest to recompute the heights by adding the divergence term to the equation. The fields thus generated were found to be similar to those obtained from the balance equation, especially in the lower levels where the divergence was small. But at 200 mb, inclusion of the divergence term raised the heights an appreciable extent. Predictably, the largest differences are found where the largest values of divergence occur. Figure 10 shows the pattern of height differences at 200 mb on September 5, at 0000Z. The maximum difference of 80 feet is located immediately downstream from the position of the incipient hurricane where the maximum divergence associated with the upper air outflow from the storm occurred.

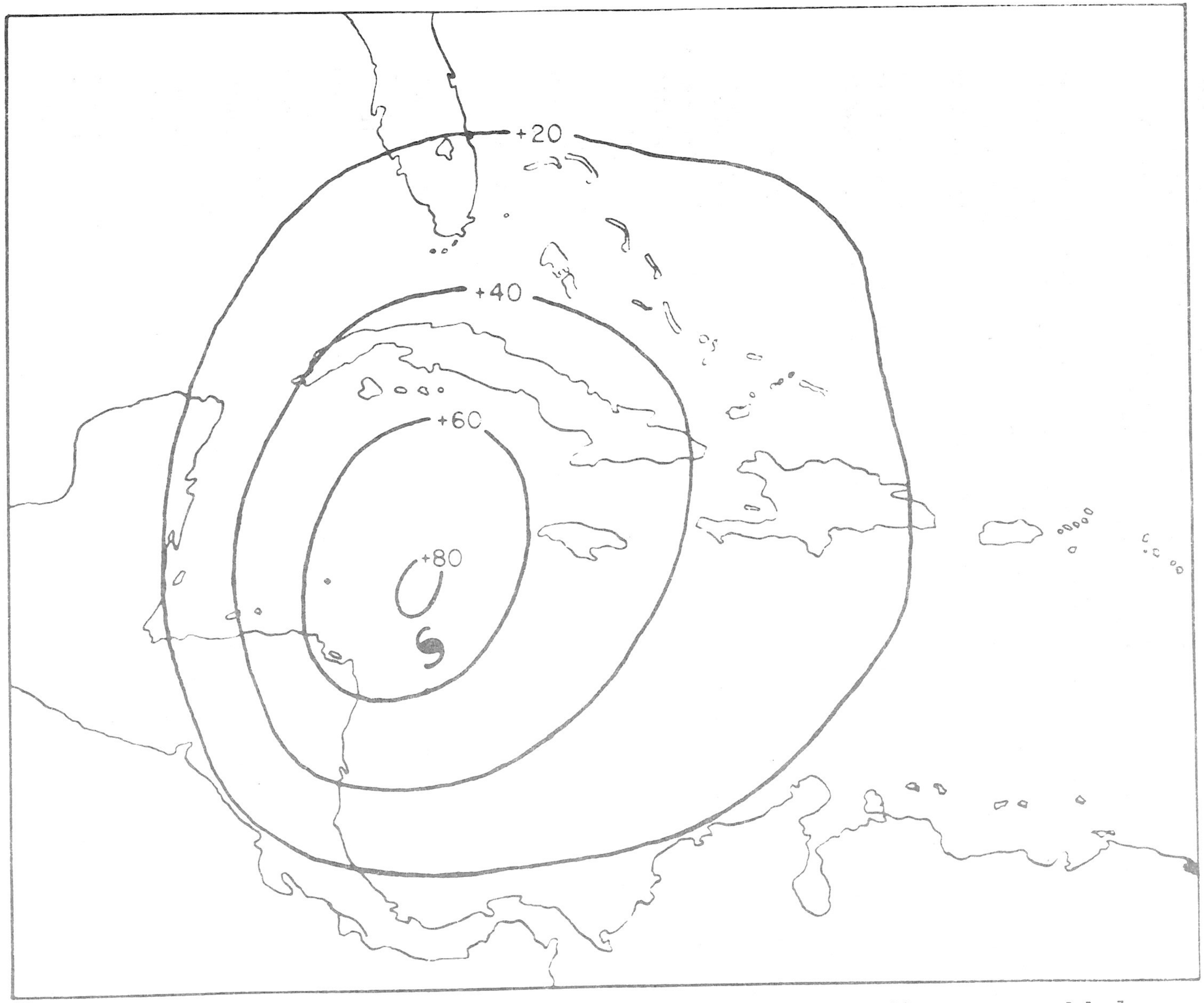

Figure 10. Differences, in feet, between 200-mb heights generated by the divergence and balance equations, September 5, 1961, 0000Z.

TABLE 1

Reported Heights Minus Heights (Feet) Generated by the Balance Equation

	P	Sept. 4, 0000Z			Sept. 4, 1200Z			Sept. 5, 0000Z		
	Mb	Mean	Median	Max	Mean	Median	Max	Mean	Median	Max
All	850	17	13	45	38	15	65	18	22	39
	500	29	19	90	33	30	90	32	18	120
Stations	200	68	58	160	47	38	174	51	36	150
Interior	850	5	8	35	19	15	65	15	15	36
	500	35	20	85	34	35	90	41	40	120
Stations	200	84	70	160	51	38	174	58	36	150
Boundary	850	20	18	45	18	18	35	21	25	39
	500	22	15	90	31	25	70	22	20	64
Stations	200	47	40	115	43	37	80	41	35	95

P = pressure

TABLE 2

Differences in Geopotential Feet Between Heights of Constant Pressure Surfaces Obtained by Any One of Six Different Types of Radiosondes and Mean Heights Obtained by the Other Five (Malet, /6/)

Type	500 Mb		300 Mb		200 Mb	
	Day	Night	Day	Night	Day	Night
1	+85	+ 6	+115	-16	+167	+ 7
2	+58	-13	+144	+69	+161	- 59
3	-95	-59	-164	-82	-203	- 79
4	+ 7	+26	+ 49	+62	+ 82	+ 89
5	+10	+57	- 33	+30	- 39	+ 33
6	-33	0	-115	-49	-154	-115

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