U.S. DEPARTMENT OF COMMERCE
NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION
NATIONAL WEATHER SERVICE
OFFICE OF SYSTEMS DEVELOPMENT
TECHNIQUES DEVELOPMENT LABORATORY

TDL OFFICE NOTE 88-5

CHARACTERISTICS OF MAP PROJECTIONS AND IMPLICATIONS FOR AWIPS-90

Harry R. Glahn

November 1988

CHARACTERISTICS OF MAP PROJECTIONS AND IMPLICATIONS FOR AWIPS-90

Harry R. Glahn

1. INTRODUCTION

The Request for Proposal (RFP) for the Advanced Weather Interactive Processing System for the 1990's (AWIPS-90) (National Weather Service, 1986) specifies several "standard areas" over which the planned Warning and Forecast Offices (WFO's) will view data and perform calculations (see NWS, 1986; for example, p. 690 and Appendix K of the SRS, pp. 1107-1175). Although these areas are given names and their sizes and locations are roughly spelled out, not all of their characteristics have been decided as of this date. Of special note in this regard is the map projection to be used for some of the areas.

For the "Hemispheric Area," which covers the Northern Hemisphere, a north polar stereographic projection is most appropriate and has been decided on. The rectilinear grids covering this area will have a gridpoint spacing in multiples or fractions of a "bedient," where the unit bedient has been defined by the National Meteorological Center (NMC) a number of years ago as 381 km at 60° N on a north polar stereographic map projection (see, for instance, Stackpole, 1978, p. 2, and also Hoke, et al., 1981, pp. 42 and 45 for an interesting historical note). Note that this unit of measure has the unusual property of changing with latitude. Strictly speaking, it also applies to a particular map projection, although it can be generalized to apply to other polar projections and as such represents the distance on that particular projection at the latitude at which the scale is quoted that corresponds to the 1-bedient distance at that latitude on a polar stereographic projection. With this definition, a 1/4-bedient grid length on any map projection <u>with scale defined at 35° N</u> is approximately 80 km at 35° N. Gridpoint data provided to the WFO's over the entire hemisphere will typically be on a 1-bedient grid, and graphics will represent that scale of information.

The "National Area" will be approximately 6500 by 7500 km. Each WFO will be able to define its National Area as to location and size, within reasonable limits. However, each such area for a WFO in the 48 states will be a portion of a larger area on a particular map projection. (National Areas for Alaska, Hawaii, and Puerto Rico will be unique to those areas and have different orientations.) For purposes of illustration, a typical National Area would be covered by a 1-bedient grid of 20 X 23 points. It has been assumed that the map projection to be used for the National Area would also be polar stereographic. Gridpoint data provided to the WFO's for their use over that area would typically be on a 1- or perhaps 1/2-bedient grid. (AWIPS-90 will be able to handle other grids, as necessary. However, it is believed that this is about the scale of information that is needed over this large an area.) Appendix K (op. cit.) states that the grid for the conterminous states would be aligned with 105° W longitude, and that grids with other orientations would be used as appropriate for Alaska, Hawaii, and Puerto Rico.

A WFO's "Regional Area" will cover approximately 1500 by 1500 km and be located with respect to its area of responsibility as it desires. This is the area over which the WFO will maintain as complete a representation as possible of current, recent past, and short range forecasts of hydrometeorological conditions. Gridpoint data will typically be at 1/4-bedient spacing (approximately 80 km at 35° N) with some surface and boundary layer information at 1/8-bedient spacing.

Quoted from Appendix K regarding the Regional Area, "One of the main precepts of AWIPS-90 is that various kinds of data, and especially the new, high density data sets, can be used together effectively to support detection, forecasting, and warning of locally severe weather and floods. It is imperative, then, that data from the next generation radars (NEXRAD) and geostationary satellite(s) (GOES) can be viewed together on the same screen and also be overlain with graphics of analyses and forecasts. This capability requires a common grid be specified at some point in the process. The common grid should be in relation to a conformal map projection that produces minimal area distortion over the total grid area. Also, a very desirable attribute of this common map projection is that north be up at every point--an attribute that may be imperative from a forecaster's viewpoint when viewing data over a very localized area around his/her station. The Lambert conformal produces little distortion, especially in mid-latitudes, but north is not everywhere up. The Mercator projection (also a conformal projection) produces more distortion, but north is everywhere up. The polar stereographic projection produces considerable distortion and north is not everywhere up. <u>The best projection to use has not been determined</u>..."

Not only must these various data be on a common map projection locally, but much data, including NMC model output and satellite data, will be transmitted from a single location, and a common map projection must be defined for this process which is the same as those used locally. Otherwise, (1) data would have to be transmitted multiple times, perhaps once for each WFO, or (2) essentially all data would have to be remapped locally. Neither of these alternatives appears to be feasible.

A WFO's "Local Area" is a nominal 750 by 750 km area positioned within its Regional Area. The same data sources supporting the Regional Area will also support the Local Area. This area is large enough to be used to maintain current mesoscale analyses and locally-produced guidance as well as official forecasts covering not only the WFO's area of responsibility but also the areas for adjoining WFO's. A mosaic of official forecasts from the responsible WFO's will be a powerful coordination tool. These mosaics will be accurate (in that no interpolation or remapping is required) and relatively easy to achieve if the Local Areas are all part of a common grid. It seems obvious that the same map projection would be used for the Local Area as for the Regional Area.

A "WFO-scale" area (or WFO Area) is not discussed in the RFP. It is now being discussed in AWIPS planning because of the realization that the nominal 460-km diameter, nominal 1-km resolution NEXRAD data must be viewed without significant degradation covering essentially a full screen. (We can consider that to be a 512 X 512 pixel area, which is close enough to 460 that at some latitude we can let 1 pixel equal 1 km. If the screen resolution is higher, say 1024 X 1024, pixel replication would be used to "blow up" the 1-km resolution data to full screen. Therefore, the WFO Area would be about 500 X 500 km, which in <u>area</u> is about half the Local Area.) Also, NEXRAD derived products are produced on a locally oriented map projection and are available at the WFO on a NEXRAD Principal User Position (PUP) in that projection. It may be that the projection used for the WFO Area will not be the same as for the Local and Regional Areas.

Three map projections are worth consideration for the Regional (and Local) areas: Mercator, polar stereographic, and Lambert conformal. Of these, the latter two could possibly be used for the National Area as well. All of these projections are conformal (which implies that the scale about a point is the same in every direction and that latitude-longitude intersections are at right angles), but north is "up" for only the Mercator. Meteorologists and hydrologists are used to viewing data over large areas on maps of various projections, but it is believed that any departure from north not being up for the very local NEXRAD data is a definite disadvantage and to be avoided if possible. However, experience might show that 460-km data on the WFO Area could be used effectively with some slight "tilt." (The 230- and 115-km diameter NEXRAD data at higher resolution would be viewed on a full screen with north up. There will not be large quantities of other data available at that high a resolution. However, it will be useful to view some data, such as mesonet and lightning strike, with this resolution radar data, even though the resolution of the lightning data is no better than a few kilometers.)

Nominal 1-km radar data are about 1 X 1 km at 57-km range and 1 X 2 km at 114-km range. Goes visible channel data will be available at a resolution somewhat less than that within, say, 80 km of the radar (depending on the location), but may have an equal or higher resolution beyond that range, provided the image registration is good. It is desirable to define the scale of the grid such that neither radar nor satellite data will have to be appreciably degraded at any latitude, and such that no more pixel replication of either type of data will be required than necessary.

This paper discusses some of the considerations regarding the possible use of the polar stereographic, Mercator, and Lambert conformal map projections for the National, Regional, Local, and WFO Areas. Attention is focussed on the "48 states;" the needs of Puerto Rico, Hawaii, and Alaska are not addressed in detail here. Also, the focus is on the WFO, although much of the discussion is probably not inappropriate for the National Centers and River Forecast Centers. Sections 2 and 3 discuss the tangent and secant conic projections, particularly as they relate to the display of NEXRAD data. Section 4 discusses possibilities for the National Area. Section 5 focuses on the Regional, Local, and WFO Areas, and finally, Section 6 contains a summary and recommendations.

2. TANGENT CONIC PROJECTIONS

The basic equations used here are given by Saucier (1955). Only enough review of Saucier's treatment is given for the results to be understandable. He states, "The process of map-making may be considered...as consisting of two steps. First, the surface of the earth is projected upon some fictitious geometric surface, the <u>image</u> surface, which is then developed by flattening into a plane surface." Consider the earth represented by the circle in Fig. 1, and a right circular cone sitting atop it with apex above (or at) the north pole. The surface of the earth (down to the equator, say) can be projected (from some point or points) onto this image surface, then the cone cut along some meridian and flattened to a plane. The result will be a portion of a circle—a pie with a slice cut out.

Two limiting cases of this "tangent" cone are (1) when the apex is at the north pole, then the cone is a plane tangent at the pole, and (2) when the apex approaches infinity, then the cone approaches a right circular cylinder tangent at the equator. The general conic projection is the basis for the Lambert conformal map, and the limiting cases are, respectively, the basis for (1) the polar stereographic and (2) the Mercator projections.

In Fig. 1 and the following sections,

ϕ = latitude,

ψ = colatitude ($90^\circ - \phi$),

ϕ_o = latitude of tangency,

ψ_o = colatitude of tangency,

σ = image scale = distance on image/distance on earth ($\sigma = 1$ at ϕ_o), and

n = geometric property of cone = Cos ψ_o.

The image scale for the conic projection is

$$\sigma = \frac{\text{Sin } \psi_o}{\text{Sin } \psi} \left[\frac{\text{Tan } \psi/2}{\text{Tan } \psi_o/2} \right]^n . \tag{1}$$

At $\phi_o = 0^\circ$ ($\psi_o = 90^\circ$), Eq. (1) represents the Mercator projection and becomes

$$\sigma = 1/\text{Sin } \psi = 1/\text{Cos } \phi . \tag{2}$$

The limiting form of Eq. (1) as ψ_o approaches 0° is

$$\sigma = 2/(1 + \text{Cos } \psi) = 2/(1 + \text{Sin } \phi) \tag{3}$$

for the polar stereographic projection.

It is noted that, in general, Eq. 1 does not represent a "perspective" projection. That is, the projection of the earth's surface to the image plane is not from a single point. However, the polar stereographic is a perspective projection, the point being on the earth opposite to the point of tangency (Bowditch, 1962).

On a conformal map, small features retain their proper shape, although the shape of large features may be distorted, and the size of two small features of equal size on the earth may not be the same on the map. It is well known that over a wide range of latitude the distortion (variation of σ) for a properly chosen Lambert is less than that for the polar stereographic or Mercator. However, all meridians are not vertical on the Lambert. If the Lambert were to be used for the WFO Area, we would want to specify a scale of 1 km/pixel (assuming a 512-pixel or similar display) at some latitude to accommodate the nominal 1-km resolution NEXRAD data.

In order to see the variation of σ and related parameters with latitude and the maximum tilts in the eastern and western 48 states for a vertical longitude of 96° W (about halfway between the easternmost and westernmost NEXRAD locations) for various tangent latitudes, a number of calculations were made for each 2.5° latitude from 0° to 90°. They can be described as follows:

A "base" latitude was defined to be the latitude at which 1 pixel = 1 km. For each of three base latitudes (25, 30, and 35 degrees), the following were computed for each of six latitudes (25, 30, 35, 40, 45, and 48 degrees) which cover the latitude belt of the NEXRAD's in the 48 states:

(1) $\sigma(B)$	= image scale of the base latitude,
(2) $\sigma(T)$	= image scale at the latitude of the NEXRAD,
(3) R(T,B)	= $\sigma(T)/\sigma(B)$,
(4) D(T,B)	= diameter of image in pixels at latitude of NEXRAD of 460-km radar data when 1 pixel = 1 km at base latitude,
(5) DIFF(T,B)	= difference in pixels of image between 512 (the assumed screen size) and D(T,B),
(6) ADD(T,B)	= the number of pixels for which 1 pixel would have to be added (+) or deleted (−) to transform 460-km data to the image of diameter D(T,B),
(7) ETILT	= tilt in degrees of easternmost NEXRAD, and
(8) WTILT	= tilt in degrees of westernmost NEXRAD.

A segment of computer printout is shown in Fig. 2 to illustrate the computations. There are three tangent latitudes represented: 0°, 45°, and 90°. (For the first and last, respectively, 0.001° and 89.999° were used to eliminate computational problems in Eq. (1). The results are the same as for the limiting equations (2) and (3), respectively.) The maximum tilts (ETILT and WTILT) vary between 0° and 28.3° as the surface is mapped (OV360) between 0° and 360°. For each tangent latitude, the image scales (SIGMA) are computed for several latitudes from the equator to the pole, those two limiting values being approximated by 0.01 and 89.99, respectively.

Then for each of the three base latitudes and for each of the latitudes representing the latitude band of the NEXRAD's, various parameters are shown. For a base latitude of 30° and a tangent latitude 0° (45°) (90°), σ at 30° is 1.15 (1.03) (1.33), and for a NEXRAD at 25° (RADAR LATITUDE), R(T,B) (SCALE RATIO) = 0.96 (1.02) (1.05), D(T,B) (CIRCLE DIAMETER) = 440 (471) (485), DIFF(T,B) (SCREEN WASTE) = 36 (20) (13), and ADD(T,B) (ADD + DROP −) = −22 (41) (18). Therefore, for the Mercator (TANLAT = .001), when the scale is 1 km/pixel at 30°, 1 pixel in 22 would have to be deleted to transform 1-km NEXRAD data at 25° latitude, whereas 1 pixel would have to be added in every 3 to transform data at 48° latitude. This results in 36 pixels on each side being wasted at 25° compared to 26 on a side at 30°. Also, 42 pixels of data on a side would be lost on the 512-pixel screen at 48° N. (No specific allowance has been made in any of the discussion concerning area of screen needed for labels, calibration scales, etc. It is assumed that this information can be put in the corners, outside the circle, or that the screen will actually be rectangular with room at the side.)

At the other extreme, for the polar stereographic (TANLAT = 89.999), when the scale is 1 km/pixel at 30°, 1 pixel in 18 would have to be added to transform 1-km NEXRAD data at 25° latitude, and 1 pixel in every 7 would have to be deleted to transform data at 48° latitude.

Note that the adding/dropping of pixels is somewhat worse for the Mercator than for the polar stereographic over the 25- to 48-degree band, and reversed. For instance, at a base of 35°, the worst loss is 1 in 10 pixels for both projections, but the polar stereographic has to add only 1 in 9 while the Mercator has to add 1 in 4—a considerable difference. Also, note that pixels are usually deleted for the polar stereographic and added for the Mercator. This is because, of course, the base latitudes are at the south side of the 25- to 48-degree latitude band. If the polar stereographic were in serious consideration for the Regional or WFO Areas, the 1 km/pixel scale would be defined further north, as it would probably be better to replicate data than to delete them.

The variation of scale for the Lambert tangent at latitude 45° is much less than for the two extremes. However, the maximum tilt is 20°—not small.

Figs. 3, 4, and 5 show some possible alternatives for the Lambert. At 10° tangent, if the scale were defined at 35°, one would need to drop 1 pixel in 16 at 25° latitude and add 1 pixel in 6 at 48°. The tilt would be only 4.9°. So for a tilt of about 5°, one has decreased the scale variation by about one third from that of the Mercator. (The range of σ from 25° to 48° is 1.10 to 1.49 for the Mercator and 1.04 to 1.28 for the 10° tangent Lambert.) Also, for the Mercator, 26 pixels would be lost at each edge at 48° and 11 for the Lambert. One hates to lose data, and 52 pixels lost out of 563 is 9%, but this is not 9% of the total data, rather it is 6%. The 11-pixel loss is only about 2% of the area (see Appendix I). Also, if the screen width happens to be larger than 512 pixels, only half that amount would be lost.

Fig. 4 shows that at tangent 17.5°, the maximum tilt is only 8.5°, and for a base of 35°, the maximum pixels dropped (added) is 1 in 25 (9). Also, essentially no data are lost at the edges. For a maximum tilt of 10.8° at tangent 22.5° and a base of 35°, a maximum of only 1 pixel is dropped (added) in 42 (11), and no data are lost at the edges. Other viable possibilities are a tangent of 17.5° and a base of 30° and a tangent of 20° and a base of 25°.

Fig. 5 shows that at tangent 25.0°, the maximum tilt is about 12°, and for a base at 35°, a maximum of 1 pixel is dropped (added) in 64 (13) and no data are lost at the edges. This does not give quite a full 512-pixel screen at any NEXRAD latitude.

A summary of some of the combinations of tangent latitude and latitude at which the 1 km/pixel resolution is specified is shown in Table 1. The resolution of GOES nominal 1-km data (at the longitude of the subsatellite point) at 25° (48°) latitude is 1.17 (1.87) km per datum, these values being taken or interpolated from information furnished by the National Environmental Satellite, Data, and Information Service (NESDIS). Appendix II furnishes more information on the resolution of the satellite data.

SECANT CONIC PROJECTIONS

The difference between tangent and secant conic projections is that the image cone is tangent to the earth in the former and cuts the earth at two latitudes in the latter. The relevant equations for the secant projection are

$$n = \frac{\log \operatorname{Sin} \psi_1 - \log \operatorname{Sin} \psi_2}{\log \operatorname{Tan} \psi_1/2 - \log \operatorname{Tan} \psi_2/2} \tag{4}$$

and

$$\sigma = \frac{\operatorname{Sin} \psi_1}{\operatorname{Sin} \psi} \left[\frac{\operatorname{Tan} \psi/2}{\operatorname{Tan} \psi_1/2}\right]^n ,$$

where ψ_1 and ψ_2 are, respectively, the northern and southern colatitudes where the cone intersects the earth. Eq. (4) cannot be used when $\psi_1 = \psi_2$, but can be used when they are different by only a very small amount. As ψ_1 and ψ_2 approach each other and ψ_0, n approaches cos ψ_0.

Fig. 6 shows an example of how certain parameters change as the cone sits (nearly) on (tangent projection at $\psi_0 = 10^\circ$), then further inside the earth, $\psi_1 = 15^\circ$, $\psi_2 = 5^\circ$ and $\psi_1 = 20^\circ$, $\psi_2 = 0.01^\circ$. Note that σ changes by about 2% at particular latitudes, but the range is 1.02 to 3.83 from 0° to 80° latitude for the tangent projection and 1.00 to 3.76 for the $\psi_1 = 20^\circ$, $\psi_2 = 0.01^\circ$ secant projection, which is no appreciable difference.

Fig. 7 is another example. The tangency latitude is 22.5°. The variation of the calculated parameters can be seen for $\psi_1 = 32.5^\circ$ and $\psi_2 = 12.5^\circ$, and for $\psi_1 = 42.5^\circ$, $\psi_2 = 2.5^\circ$. There may appear to be a slight advantage for the secant projection. For example, the circle diameter in pixels varies less between 25° and 48° for the secant than for the tangent, but only by 2--about a 4% improvement. This has been achieved, however, by a slightly increased maximum tilt, a difference of 0.3° or a 3% degradation.

The scale of a map can be quoted at any latitude regardless of the projection. That is, for a secant Lambert, the scale could be quoted at some latitude between ψ_1 and ψ_2, and chosen properly the scale would vary in the same direction for all other latitudes. Similarly, for a tangent Lambert, the scale could be quoted for two latitudes, one above and one below ψ_0, since for any ψ_0 there are matching latitudes where the scales are equal. (Note, however, that for, say, $\psi_0 = 45^\circ$, the "matching scale" latitude of 30° is not 60°.) There is a popular misconception that the secant projection is really "better" than the tangent projection. In fact, they are equivalent! The only difference is where the scale is quoted.

If we are free to chose two of the three colatitudes ψ_0, ψ_1, and ψ_2, then the other can be computed by setting

$$n = \text{Cos}\ \psi_0 = \frac{\log \text{Sin}\ \psi_1 - \log \text{Sin}\ \psi_2}{\log \text{Tan}\ \psi_1/2 - \log \text{Tan}\ \psi_2/2} . \quad (5)$$

The image scale of a tangent projection (call it σ_0) will be unity at the point of tangency. The image scale of the secant projection (call it σ_1) will be unity at the points of secantcy. Therefore, they will not be equal for the two projections, but the projections will be equivalent if σ_0 / σ_1 is a constant for all ψ.

$$\frac{\sigma_0}{\sigma_1} = \frac{\dfrac{\text{Sin}\ \psi_1}{\text{Sin}\ \psi}\left[\dfrac{\text{Tan}\ \psi/2}{\text{Tan}\ \psi_1/2}\right]^n}{\dfrac{\text{Sin}\ \psi_0}{\text{Sin}\ \psi}\left[\dfrac{\text{Tan}\ \psi/2}{\text{Tan}\ \psi_0/2}\right]^n}$$

$$= \frac{\text{Sin}\ \psi_1}{\text{Sin}\ \psi_0}\left[\frac{\text{Tan}\ \psi_0/2}{\text{Tan}\ \psi_1/2}\right]^n$$

does not depend on ψ, so the projections are equivalent. As an example, for $\psi_0 = 45^\circ$, $\psi_1 = 60^\circ$, and $\psi_2 = 31.235^\circ$, n = .07071 and $\sigma_0/\sigma_1 = 0.9684$. It is also interesting to note that for $\psi_1 = 60^\circ$ and $\psi_2 = 30^\circ$, the "equivalent" tangent projection is not $\psi_0 = 45^\circ$, but rather 44.31°, as evaluation of Eq. (5) shows. That is, for this secant projection, σ is a minimum at 44.31°.

Since there is no difference in the distortion between the tangent and secant projections, we should prefer the tangent because of its ease of use in mathematical calculations. The geometric property of the cone, n, which must be used in all calculations involving map factors or transformations from the Lambert to another projection (or vise versa) is simpler for the tangent than for the secant. The secant projection is essentially a "two parameter" north polar projection, while the tangent is a "one-parameter" projection.

4. THE NATIONAL AREA

As stated previously, a WFO's National Area will cover approximately 6500 X 7500 km. It would be desirable to use the same map projection for all areas, then a user wouldn't have to change her/his perspective, or view of the geography, whenever she/he switched from one viewing area to another. This, of course, is not practical. The polar stereographic is ideal for the Hemispheric Area, but unusable for the WFO Area. Then the question is whether the National Area should use the polar stereographic and match the projection for the Hemispheric Area or use the same Lambert, if indeed the Lambert is to be used, and match the Regional and Local Area projections.

According to Fig. 1 in Appendix K (op. cit.), a typical National Area will encompass essentially all of North America and might range from 20^o to 87^o N. Over this range of latitude, 20^o to 87^o, σ for the polar stereographic ranges from 1.49 to about 1.00. The Mercator is, of course, unusable that far north. For the Lambert tangent at 45^o, σ ranges from 1.09 at 20^o to 1.00 at 45^o to 1.92 at 87^o, which would be a possibility, especially if we could limit its use beyond, say, 83^o where $\sigma = 1.50$. It's interesting to note that over this range of latitude (20^o to 87^o) the polar stereographic has less range of σ(1.49 to 1.00) than does the 45^o tangent Lambert (1.00 to 1.92). Over 20^o to 83^o, the range of σ is about equal--1.49 to 1.00 for the polar stereographic and 1.00 to 1.50 for the Lambert. The change with latitude is less rapid on the Lambert, even with the minimum at 45^o, than for the polar stereographic, except north of about 60^o.

However, the maximum tilt for a 45^o tangent Lambert (20.0^o) is probably unacceptable for the display of radar data, as we have seen before. Rather, better possibilities are for a Lambert tangent at some lower latitude. For a 25^o tangent, σ ranges from 1.00 at 20^o to 4.49 at 87^o. Even at 83^o, σ is rather large--2.76. The picture is even worse for the 10^o tangent Lambert; σ ranges from 1.02 at 20^o to 5.13 at 83^o to 10.31 at 87^o. Even at 70^o, σ is as high as 2.00. Clearly, a 10^o tangent Lambert is not usable for the National Area from the variation of scale alone. Even a 25^o tangent Lambert doesn't seem to be a good choice. It should also be noted that the National Area for Alaska extends all the way to 90^o latitude, and it is desirable to have all National Areas on the same projection.

Fig. 8 shows a portion of what would be a National Area on a 25^o tangent Lambert. The latitudinal extent is from 20^o to 80^o, whereas the National Area, as stated previously, is planned to go as far north as 87^o. This shows graphically the change in σ from one between 20 and 25 degrees to about two between 75 and 80 degrees. If the longitude lines were extended to the pole, as they would be for the Alaskan National Area, there would be no "map" on the other side of the pole. This is due to the slice cut from the pie (see Section 2), in this case, the slice being greater than half the pie for the low tangent latitude. Also, in the northern parts of the rectangular map, there would be no data unless the same position on the earth were represented more than once.

Another consideration is that most of the numerical model output to be furnished by NMC over the National Area, and over the Regional and Local areas as well, is tied to a rectilinear grid related to a polar stereographic projection. Gridpoint and graphic products furnished and used on a grid related to that same projection can retain the information initially in the model data on which they were based. (It may be that interpolation to a different array of points will be necessary or that some gridpoint products will not be sent at full resolution, but the latter at least is a decision that can be made based on requirements. Graphics can retain whatever level of detail seems appropriate at all latitudes.) If a different map projection is used and a transformation is made, either (1) some information will be lost other than that resulting from just the possible interpolation to another set of points on the same projection, or (2) to retain all information at all latitudes, the gridpoints will have to be at least as dense at all latitudes on the receiving grid as on the donor grid. This latter would increase product size at least for gridpoint products. It's also likely that graphics would be produced by first preparing a gridpoint field on the same map projection as the graphic to be produced. This shouldn't be necessary if the same map projection were used for the graphics as for the model output.

As an example, suppose that a 1-bedient grid exists relative to a polar stereographic projection covering the latitudes 20^{o} to 87^{o}. The gridpoint spacing at 20, 35, 48, 60, and 87 degrees is, respectively, 274, 321, 356, 381, and 408 km. If we want to transform this to a Lambert tangent at 25^{o} and not lose information at the smallest spacing (274 km), we would have to make the spacing on the Lambert 274 km at 20^{o}. Then, the spacing at latitudes 20, 35, 48, 60, and 87 degrees would be, respectively, 274, 269, 245, 217, and 61 km. It is seen, then, that in order to retain the information at 20^{o}, many more grid points would be necessary to cover the National Area or any portion thereof.

Alternatively, we could "match" the spacing at 60^{o} and let the spacing be 381 km on the Lambert there. Then the spacing at 20, 35, 48, 60, and 87 degrees would be 480, 471, 449, 381, and 107 km, respectively. This would be degrading the information south of 60^{o} and still require more than enough points north of 60^{o} (although the proportion of points north of 60^{o} is rather small). At 35^{o}, the model would produce data every 321 km (on a 1-bedient grid), but it would be transmitted degraded to one point every 471 km, almost 50% larger. This mismatch (and, in fact, general reversal) of scale may not be a big factor, since we may not transmit gridpoint data over the National Area at the computational spacing anyway, but shouldn't be entirely neglected.

(The possible mismatch of scales for the Regional and Local Areas is also of some concern, but other factors play a bigger role there. NMC model data are still of great use, but locally produced data, such as from mesoanalyses and local models, and fine scale data not intrinsically related to the polar stereographic grid play an increasingly greater role in the progression from National to Regional, Local, and WFO Areas. Adequate attention must be given, though, to retaining the useful information on whatever projection is used.)

In performing certain computations on a grid, and especially those associated with numerical models, a "map factor" (see Appendix III) must be used. This is rather simple for the polar stereographic. When the scale is defined at 60° N, the map factor is

$$m = \frac{1 + \sin 60^\circ}{1 + \sin \phi} = \frac{1.866}{1 + \sin \phi} .$$

Even the Mercator has a simple expression for map factor:

$$m = \frac{\cos 60^\circ}{\cos \phi} = \frac{0.5}{\cos \phi}$$

for a scale defined at 60°. But the Lambert, tangent at 25° and with its scale defined at 60°, has a slightly more imposing map factor:

$$m = \frac{\sin 30^\circ}{\sin \psi} \left[\frac{\tan \psi/2}{\tan 30^\circ/2}\right]^{\cos 65^\circ} = \frac{0.87(\tan \psi/2)^{0.423}}{\sin \psi}$$

So, computations involving a map factor are slightly more complicated on the Lambert than the Mercator or polar stereographic, but this is trivial compared to other considerations, especially since most local uses will not involve the map factor.

Satellite data composited from both GOES EAST and GOES WEST are to be furnished to the WFO's each 30 minutes on the Hemispheric Area. It may be very desirable to view these data overlaid on National Area products. If (1) the map projections for the Hemispheric and National Areas are the same, (2) the grids for them are aligned along the same longitude, and (3) the grid spacing of one is equal to 2^n of the other, where n is an integer, then it may be a fairly simple matter to arrange for transporting a portion of a Hemispheric Area satellite data composite to the National Area without further degradation of the data. If different projections are used, the process is still possible, but is more complicated and computer intensive, and there will be data degradation.

The National Area will be used by the forecaster predominately to get an overall, or "national", view of the weather patterns over the United States and adjacent regions. This view is obtained in the context of the Hemispheric Area charts made available to the WFO. The primary "work areas" for the WFO will be of smaller size where data of higher resolution are available. Because of the anticipated uses of the data on the various areas, and because of the factors discussed in this section, the polar stereographic appears to be the best projection for the National Area, as assumed previously.

5. THE REGIONAL, LOCAL, AND WFO AREAS

Available to the WFO's will be a radar summary chart and a radar wind chart distributed from NMC every 30 minutes. The radar summary chart will be prepared from radar coded messages (RCM's) sent from the WFO's containing data at a nominal 10-km resolution related to the Limited Area Fine Mesh (LFM) model, polar stereographic grid. The wind chart will be prepared from NEXRAD Velocity Azimuth Display (VAD) winds sent in the same message. Appendix K (op. cit., p. K-52) states that the radar summary chart will be on a Mercator-oriented grid, which is the projection used for sizing and illustrative purposes for the Regional Area in Appendix K. This would require a remap at NMC.

Only data which have been quality controlled by a forecaster will be used in the radar summary chart, so some data gaps may exist due to lack of time at the

WFO's, particularly at one of the two times per hour for which the NWS is not initially requiring that manual quality control be done. The AWIPS RFP as presently written does not require that the RCM's be available to other WFO's (NWS, 1986, pp. 437-438). However, it seems that the data in these messages would be very useful to the WFO's, and the associated transmission volume is not large. If the RCM's were made available to surrounding WFO's, each WFO could composite its own 10-km radar map over its Regional or Local Area and use all the data, not just edited, as desired. In addition, RCM's from Department of Defense (DoD) NEXRAD's containing unedited data that will not be in the radar summary chart could be made available and used in the composite at nearby WFO's. These data, related to a polar stereographic projection, will have to be transformed to the Regional Area projection; this should be a minor processing load. The data will be further degraded by the transformation, but should be adequate for diagnosing synoptic and subsynoptic scale features. This locally produced chart for Regional and Local Area use should be quite complete and serve a very useful purpose. A longer-term goal would be for NEXRAD to produce the RCM's in the Regional Area projection, then the necessary remap would be done at NMC (once for each message) rather than at the WFO's (once at <u>each</u> WFO that uses it).

A primary purpose of the centrally provided radar summary chart and the VAD wind chart <u>could</u> be to overlay them with National Area products to help with the "overview" process and thereby be a part of the Synoptic Overview Package (NWS, 1986, p. 690). If it is assumed that a WFO <u>will</u> produce 10-km resolution charts for its Regional or Local Area, then use of the radar summary chart would be essentially limited to National Area use and, consequently, should be formed and transmitted on the LFM-oriented polar stereographic grid (assuming, of course, this polar stereographic <u>is</u> used for the National Area). This has the advantage of not requiring a transformation to another grid at NMC and the resulting degradation of information.

NEXRAD data, other than the radar coded message, will be in a "flat earth" system with the station at the center (essentially, polar coordinates or those coordinates just transformed to Cartesian coordinates). Over an area as small as the radar umbrella it doesn't matter a great deal whether the map projection used for the geography is flat earth, a locally-oriented Lambert conformal, a local stereographic, or some other locally-oriented projection. All such radar data should be able to be viewed full screen with north up. Since the NEXRAD data arrive in the flat earth projection, they can be viewed that way. However, arrangements must be made for overlaying other data.

Quite likely, although experience with such data is limited, the nominal 1-, 2-, and 4-km NEXRAD data will need to be regularly overlaid with other products, such as 1- to 2-km visible and 4- to 8-km infrared satellite data and mesoanalyses based on data a few kilometers apart. This can be done on the Local Area [where some minimal amount of data from adjacent radars will also be available, at least when the adjacent WFO is in the Warning Mode of operation, at about that same resolution (NWS, 1986, pp. 442, 611, 612, 927)] provided the radar data are transformed to the Local Area projection. The RFP (NWS, 1986, p. 608) specifies that a 4-km composite reflectively mosaic shall be created when the WFO is in the Alert or Warning Mode using data from up to eight non-collocated NEXRAD's. Overlaying could also be done on the WFO Area, provided all such data were on the WFO Area projection. Even on the WFO Area, some adjacent radar data could be used to fill "holes" in the collocated NEXRAD coverage, to fill in the "corners" of the screen, or just where the adjacent radar might be closer to a point in the area than the collocated NEXRAD. Experience will undoubtedly define the need and eventual availability for such data on a site by site basis.

On the other hand, 1/4- and 1/2-km resolution data may usually be used by themselves. Table B.4.3 of the SRS (NWS, 1986, Appendix F, p. F-69) specifies that 1/4-km data will be stored by AWIPS only over a 120 X 120 km area and that 1/2-km data will be stored only over a 230 X 230 km area. There is good reason for this--for each nominal resolution, the data are not of that high a resolution at larger distances. Satellite data in the visible channel are of somewhat comparable resolution to the 1-km radar data (see Appendix II), so their detail does not match the 1/4- and 1/2-km radar data. It appears that these high resolution radar data, used by the forecaster primarily in severe convective weather situations, will serve their purpose with possibly only a very few products available for overlaying. (Note that at 1/4 km/pixel, data over 120 km will nearly fill a 512-pixel screen.) The capability to transform products to the NEXRAD "polar coordinate" projection should be available, but the expectation would be for only limited use of the capability unless a polar coordinate system is used for the WFO Area. For instance, satellite data would be overlaid with the 1-, 2-, and 4-km radar data predominately on the WFO and larger Areas.

The question remains, "What projection(s) should be used for the Regional and Local Areas and for the WFO Area--should they be the same or different?" A key to the answer is whether <u>any</u> "tilt" (departure of north from up on the screen) can be tolerated on these areas. That question can probably not be answered without extensive testing in different parts of the country. Forecasters in the West and the East are already used to considerable tilt to most of their maps, but <u>not</u> when viewing radar data and preparing warnings from them.

Compromises must, of course, be made. A Mercator has north up, but the variation in scale is considerable and the necessity to add and/or drop pixels from the 1-km radar data to get them to a common projection (necessary for overlaying centrally distributed products and for use at other WFO's) degrades, to some extent, the data. Also, at some latitudes, the screen would not be quite filled on the WFO Area, and at other latitudes, data would be lost at the edge (a screen size different from 512 X 512 may alter this statement.)

The Lambert tangent at some low latitude (lower than we are used to seeing for such a map) offers a compromise. Its scale will vary with latitude also, and in the same direction as the Mercator, but the trade-off is increasing tilt for smaller scale variation. Not only does the lesser scale variation help in not having to add/drop pixels for the radar data, but the "mismatch" between the polar stereographic grid on which the NMC models are run is less severe than with the Mercator (see the discussion in Section 4). Also, less replication need be done to get satellite data onto the common grid.

Looking again at Table 1, we see that at tangents $\geq$ 22.5 degrees, the maximum tilt is $\geq$ 10.8 degrees, and the screen is never quite full. On the other hand, very few pixels are dropped and relatively few added. For the Mercator, at least 26 pixels are lost at each edge at 48°, which is 6.6% of the area, and 1 in 10 pixels are dropped at 25° and 1 in 4 are added at 48°. If the scale is defined even farther north than 35°, so that less pixels are lost at the edge at northern latitudes, the loss of data at 25° (dropping of pixels) will be even greater.

If we compare the Mercator with the 17.5° tangent Lambert, both with scale defined at 25°, the Mercator has an unacceptable data loss (56 pixels at each edge) at 48° and a replication rate of 1 in 3, while the Lambert has an acceptable data loss (12 pixels at each edge) at 48° and a replication rate of 1 in 6, half that of the Mercator. But the Lambert's maximum tilt is 8.5°. A tilt of 8.5° may seem like a lot. Table 2 shows the approximate percent of the area of the 48 states with less than specified tilt angles for the 17.5° Lambert.

So, even with a maximum tilt of 8.5°, nearly 70% of the country would have 5° tilt or less. According to information (consisting of the climatic frequencies of tornadoes by state over the past 38 years) furnished to the NWS National Verification Committee by the National Severe Storms Forecast Center (NSSFC), only about 6.0% of the tornadoes occur in areas with greater than 5° tilt.

In Table 1, the tangent of 36.8° gives the minimum distortion between 25° and 48°, and is equivalent to a Lambert secant at 25° and 48°. There is no need to consider for Local Area use a tangent above 36.8°. Little distortion occurs between those latitudes, and adding and dropping pixels is negligible. With a scale of 1 km/pixel at 35°, the scale is 0.98 km/pixel at 25° and 48°. The drawback is the maximum tilt of 16.9 degrees. Of lesser importance is the fact that the 512-pixel screen is never quite full. If this tilt could be tolerated, the scale could be modified to, say, 0.9 km/pixel. Then there would be a replication of about 1 pixel in 10 at all latitudes between 25° and 48°, and there would be a 4-pixel loss at each edge. The screen would be essentially full at all latitudes. However, this change in scale would require the same change in scale for the nationally produced satellite products targeted for Regional Area use. Satellite data are already being replicated, and this would require even more replication. Also, the product size, which is already large and highly influences the data transmission rate, would be increased by several percent.

Figs. 9 through 13 show examples of possible projections for the Regional Area. On each, longitude 95° W is approximately vertical, and circles of 460-km diameter are shown at latitudes 25°, 35°, and 48°. The circles on all figures are the same size at 35° N, where the scale might be defined as 1 km/pixel. Fig. 9 shows the Mercator; the area transmitted from NMC needs to go to about 57° N, which is just above the top edge of the map as shown. The dimensions of the Local Area would be about 750/460 = 163% of the radar circle diameters. The difference in circle variation with latitude decreases, of course, as the latitude of tangency goes from 0°, to 10°, etc. to 36.8°, at which point they are the same size as near as the eye can tell. The maximum tilt in Fig. 12 is 12°.

6. Summary and Recommendations

A decision must be reached soon on what map projections and basic gridpoint intervals should be used for the National, Regional, Local, and WFO Areas for AWIPS-90. There is consensus that the polar stereographic with a basic interval of one bedient (381 km at 60° N) oriented with a vertical along 105° W be used for the Hemispheric Area; this is the projection and roughly the scale used for models at NMC that will be producing products for this area and is the orientation now used on similar AFOS products.

For reasons cited herein, the best choice for the National Area projection seems to be the polar stereographic with a basic interval of one bedient oriented with a vertical along 105° W; this is the projection and orientation used at NMC for many purposes, including the LFM model. The Nested Grid Model (NGM) uses that projection and orientation, although the grid interval is not in even fractions of a bedient. Since grids of future models may have other grid spacings, one choice may be as good as another in that regard. It should be noted that NEXRAD is on the way to producing a radar coded message based on the LFM grid. A major factor in choosing the polar stereographic for the National Area is that it extends to the north pole for Alaska and nearly there for the 48 states. Use of the polar stereographic also leaves open the possibility that the area could be extended beyond the pole.

A conic projection is the basis for the Lambert conformal projection and in limiting cases for the Mercator and polar stereographic. Each of the three can be either tangent or secant projections. However, for every north polar secant projection there is an equivalent north polar tangent projection and visa versa. Although the image scales are not the same (the projection distances are different), once the scale is defined and the map factor applied, the results are the same. Therefore, secant versus tangent projections should not be an issue. However, the tangent projection is easier to deal with mathematically than the secant, and, therefore, should be preferred for use in AWIPS-90.

It still seems unlikely that major remapping will occur from the centrally provided gridpoint, graphic, and satellite data to some other projection. For one reason, this would be a major processing load and for another it would in many cases constitute <u>another</u> remap. For instance, satellite data will already be remapped to some grid system other than the "satellite view." The mapping from the satellite view to something else is the most computationally intensive one and must be done centrally. Remapping capability must be available at WFO's, but computer resources will limit its use.

There are definite advantages in having the Regional, Local, and WFO Areas on the same projection. If the 1-km satellite and radar data can be used predominately on that projection, a minimum of data transformation will be necessary. All NEXRAD data will have to undergo <u>some</u> processing to get them from the NEXRAD formats to the AWIPS database. The 1/4- and 1/2-km data, when needed, can be used on the projection of receipt, and the 1-, 2-, and 4-km data can be put on the WFO/Local/Regional Areas projection. The ability to display either the 1/4- to 1/2-km or the 1- to 4-km data on the other projection must be provided, but would be needed for only a small percentage of the total available products. It appears a Lambert conformal tangent somewhere between 17.5° and 36.8° with a scale of 1 km/pixel defined at 35° N is a good choice for the Regional, Local, and WFO Areas. If a Lambert is used, it will probably be necessary and desirable to transmit separate gridpoint and graphic products from NMC for the 48 states and Puerto Rico, rather than combined products as stated in Appendix K (op. cit., pp. K-5, K-16). Appendix IV offers some information on this topic. There is another issue concerning the Puerto Rico Regional Area; the NGM "C" grid does not adequately cover it. A decision to use a Lambert projection should not be irrevocable, even though such things tend to be difficult to change, once implemented. We should insist on software at NMC and NESDIS and at the WFO's flexible enough to handle a change from a chosen Lambert to, say, another Lambert or a Mercator if the need arises.

If a Lambert is chosen for the Regional Area, any graphic and gridpoint data sent from NMC to support that area should be on that projection. Gridpoint data should be sent interpolated to the grid defined for the area. This will relieve the WFO's of that chore, and we do not want to encourage the use of grids on-station other than those defined for routine use by routinely sending data on "original" model grids. By defining the 1/4-bedient gridlength as far south as 35° N, the useful detail is preserved. Interpolation at NMC will undoubtedly be biquadratic (or some similar process), so that the gridpoint values should be quite representative of the model. The NGM "C" grid has a resolution of approximately 1/4 bedient. Therefore, data sent for the Regional Area with an 80-km spacing at 35° N will have approximately the resolution of the NGM. Many contour routines calculate contour "crossings" between gridpoints by linear interpolation. Therefore, retention of the model information by quadratic interpolation to another set of points is probably better than that of time-efficient contour routines. Producing a graphic on one map projection and transforming it to another projection would not be an inexpensive process.

Appendix K (op. cit., pp. K-9, K-17) defines Eastern and Western CONUS Sectors for satellite data. The Eastern Sector extends westward to 105° W--30° from GOES EAST longitude; the Western CONUS Sector extends eastward to 90° W--45° from GOES WEST longitude. Appendix II discusses the resolution of the nominal 1-km satellite data away from the subsatellite point. Longitude 105° W is halfway between the planned 75° and 135° positions for GOES EAST and WEST, respectively, and, therefore, the resolution of the data from both satellites is the same there. At 30° N (48° N), the long dimension of the viewed rectangle increases from 1.6 to 2.1 (2.3 to 3.2) for GOES WEST from 105° to 90°W (see Fig. 18). Since such quantitative resolution figures were probably not readily available when the sectors were defined, it seems appropriate to again review the requirements for these specific areas. Perhaps the Eastern Sector should extend slightly westward and the Western Sector decrease in size along the eastern boundary.

In summary, my recommendations are:

1. Continue to plan for the polar stereographic projection oriented with a vertical along 105° W for both graphic and gridpoint data and a basic unit of one bedient for the Hemispheric Area.

2. Use the polar stereographic oriented with a vertical along 105° W for both graphic and gridpoint data and a basic unit of one bedient for the National Area.

3. Initially, use a Lambert conformal tangent at 25° N oriented along 95° W with a basic unit of exactly 80 km (approximately 1/4 bedient) at 35° N for the Regional, Local, and WFO Areas. This gives a maximum tilt of 13° in the West and 11° in the East; about 70% of the central part of the country would have a tilt of 7 degrees or less. The 95° orientation is a slight adjustment from the more central 96° toward higher density of severe storms and population concentrations. Also, 95° is a "nice, round" figure, many maps depict 95° longitude lines, and it just happens to go smack-dab through Joe Schaffer's back yard. Gridpoint data should be sent from NMC already interpolated to this grid to relieve the WFO's of that chore. Quite likely, graphics will not be sent in quantity from NMC for the Regional Area, but when graphics are sent, they should be in relation to this Lambert grid.

4. Assure that the software for generation of all data products at the central location be flexible enough so that a change from the Lambert used initially to another Lambert or Mercator can be easily made.

5. Assure that software at WFO's key on the product/data identifying information sent with the product and be able to handle a change from the Lambert initially used to another Lambert or to a Mercator with no change in software necessary at the WFO.

6. Use a flat earth projection to view radar data as received from NEXRAD in a north-up orientation. It is expected that only a few products would be remapped to this projection, although the remapping capability must exist. As data are received from the collocated NEXRAD, store the 1/4- and 1/2-km resolution data in the AWIPS database on the projection of receipt, and remap the 1-, 2-, and 4-km data to the WFO Area. This, together with the Regional, Local, and WFO Areas all being on the same projection, should greatly facilitate the overlaying of different types of data, except the high volume, high resolution (1/4- and 1/2-km) radar data would not normally be overlaid with other courser-resolution data, such as NMC graphics. The fine scale radar data (not available beyond 57-km from the radar site for the 1/4-km data and 115-km for the 1/2-km data) would be used to "zero-in" on specific areas of interest (when within the range of the data). Some data such as mesonet and lightning strike can be remapped to that same projection.

7. Provide the radar summary chart over the appropriate portion of the polar stereographic National Area product for use with the Synoptic Overview Package. This will, then, not require a remap at NMC. Also, make available to each WFO the RCM's from surrounding WFO's and DoD sites so that it can composite the 10-km radar data over its Regional Area. This recommendation is based in part on the expectation that some of the radar data in the RCM's from WFO's will not be manually quality controlled, and, therefore, will be missing from the radar summary chart, and also that RCM's from DoD radars will not influence the radar summary chart.

8. Continually evaluate the choices made for initial implementation as AWIPS-90 is installed across the country. If it appears better choices are available, experimentation with them can be done relatively easily (compared to now) with the hardware and software then in place, and a change implemented if appropriate.

9. The use of a Lambert projection for the Regional Area will probably require a separate product be transmitted from NMC for Puerto Rico. Once the decision is made for the Conterminous U.S. Regional Area projection, the exact areas for the products for the 48 states and Puerto Rico must be defined. Mercator for Puerto Rico's Regional Area is appropriate. However, the NGM "C" grid does not cover Puerto Rico's Regional Area adequately, so which data to provide must also be agreed upon.

10. An analysis should be done for Alaska and the map projections finalized for use there. The areas for Hawaii as defined in Appendix K (op. cit.) are appropriate.

11. In connection with defining the exact areas over which satellite data are to be transmitted from a central location, revisit the "overlap" portions

of the Eastern and Western CONUS Sectors, as defined in Appendix K (op. cit. p. K-9); determine whether these areas best meet the requirement for satellite data. Also, it must be assured that the grids used by NMC and NESDIS are coordinated; that is, we must not only agree on the same map projection and orientation, but also that the grids are the same. This does not imply that data have to be transmitted at every gridpoint for every product. The use of a Lambert rather than Mercator projection should decrease the product size for the nominal 1-km satellite products somewhat; alternatively, products could be sent over larger regions.

ACKNOWLEDGMENTS

A number of persons furnished me information relating to the preparation of this document, including Bob Elvander of the NEXRAD JSPO, Paul Jendrowski of Unisys Corporation, Bob Saffle of TDL, Denny Walts and Dan Binkerhauer of PROFS, Jamison Hawkins, Kathy Kelly, and Gene Legg of NESDIS, Bob Morris of General Sciences Corporation, Jim McDonell and John Stackpole of NMC, Paul Polger and Mary Heffernan of OM, and Jack Jalickee of OSD. Wendy Wolf of TDL contoured on AFOS the satellite data "expansion factors" shown in Figs. 15 through 19. Elliott Jacks and Ron Anderson did the drafting. To these people, I am grateful. I especially thank Dave Shimomura of NMC for producing the maps shown in Figs. 8 through 13 on his "magic machine." The responsibility for the material as presented and for the conclusions and recommendations, I must accept.

REFERENCES

Bowditch, N., 1962: American Practical Navigator. U.S. Navy Hydrographic Office, United States Government Printing Office, pp. 69-88.

Gerrity, J. P., 1973: On map projections for numerical weather prediction. NMC Office Note No. 87, National Weather Service, NOAA, U.S. Department of Commerce, 11 pp.

Hoke, J. E., J. L. Hayes, and L. G. Renninger, 1981: Map projections and grid systems for meteorological applications. AFGWC Technical Note AFG-WC/TN-79/003, Air Weather Service, MAC, USAF, Department of Defense, 86 pp.

National Weather Service, 1986: Request for Proposal, Solicitation No. WE3400-7-00002, National Oceanic and Atmospheric Administration, U.S. Department of Commerce, 1414 pp.

Saucier, W. J., 1955: Principles of Meteorological Analysis. The University of Chicago Press, Chicago, pp. 29-38.

Stackpole, J. D., 1978: The National Meteorological Center's operational seven layer model on a northern hemisphere Cartesian 190.5 km grid. NMC Office Note No. 177, National Weather Service, NOAA, U.S. Department of Commerce, 8 pp.

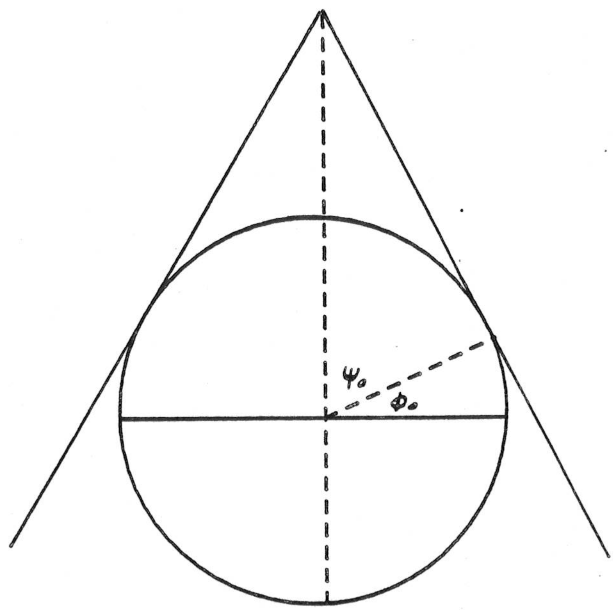


Figure 1. Tangent cone as a basis for a conic projection.

TANLAT	ETILT	WTILT	CTLAT	NCONE	OV360
.001	.0	-.0	90.00	.000	.0

LATITUDE	.01	10.00	20.00	25.00	30.00	35.00	40.00	45.00	48.00	50.00	55.00	60.00	65.00	70.00	80.00	89.99
SIGMA	1.00	1.02	1.06	1.10	1.15	1.22	1.31	1.41	1.49	1.56	1.74	2.00	2.37	2.92	5.76	******

	BASE LAT = 25.0				BASE LAT = 30.0				BASE LAT = 35.0			
RADAR LATITUDE	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)
25.0	1.00	460	26	****	.96	440	36	-22	.90	416	48	-10
30.0	1.05	481	15	21	1.00	460	26	****	.95	435	38	-18
35.0	1.11	509	2	9	1.06	486	13	17	1.00	460	26	****
40.0	1.18	544	-16	5	1.13	520	-4	8	1.07	492	10	14
45.0	1.28	590	-39	4	1.22	563	-26	4	1.16	533	-10	6
48.0	1.35	623	-56	3	1.29	595	-42	3	1.22	563	-26	4

TANLAT	ETILT	WTILT	CTLAT	NCONE	OV360
45.000	19.9	-20.0	45.00	.707	254.6

LATITUDE	.01	10.00	20.00	25.00	30.00	35.00	40.00	45.00	48.00	50.00	55.00	60.00	65.00	70.00	80.00	89.99
SIGMA	1.32	1.18	1.09	1.06	1.03	1.01	1.00	1.00	1.00	1.00	1.02	1.04	1.08	1.13	1.36	10.19

	BASE LAT = 25.0				BASE LAT = 30.0				BASE LAT = 35.0			
RADAR LATITUDE	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)
25.0	1.00	460	26	****	1.02	471	20	41	1.04	480	16	23
30.0	.98	449	31	-42	1.00	460	26	****	1.02	468	22	56
35.0	.96	441	35	-24	.98	452	30	-57	1.00	460	26	****
40.0	.95	436	38	-20	.97	447	32	-36	.99	455	28	-93
45.0	.95	435	39	-18	.97	445	33	-32	.99	453	29	-69
48.0	.95	435	38	-19	.97	446	33	-33	.99	454	29	-77

TANLAT	ETILT	WTILT	CTLAT	NCONE	OV360
89.999	28.2	-28.3	.00	1.000	360.0

LATITUDE	.01	10.00	20.00	25.00	30.00	35.00	40.00	45.00	48.00	50.00	55.00	60.00	65.00	70.00	80.00	89.99
SIGMA	2.00	1.70	1.49	1.41	1.33	1.27	1.22	1.17	1.15	1.13	1.10	1.07	1.05	1.03	1.01	1.00

	BASE LAT = 25.0				BASE LAT = 30.0				BASE LAT = 35.0			
RADAR LATITUDE	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)
25.0	1.00	460	26	****	1.05	485	13	18	1.11	509	2	9
30.0	.95	436	38	-19	1.00	460	26	****	1.05	483	15	20
35.0	.90	416	48	-10	.95	438	37	-21	1.00	460	26	****
40.0	.87	398	57	-7	.91	420	46	-12	.96	441	36	-24
45.0	.83	383	64	-6	.88	404	54	-8	.92	424	44	-13
48.0	.82	375	68	-5	.86	396	58	-7	.90	415	48	-10

Figure 2. Segment of computer printout showing data for the conic map projection at tangents near 0^{o}, 45^{o}, and near 90^{o}. For each tangent latitude, the image scale σ is given for several latitudes. Then, for each of three base latitudes (B) and for each of six latitudes of radar sites (T), the scale ratio R(T,B), the diameter of a radar circle in pixels for a diameter of 460 km at the base latitude, the number of pixels wasted at each edge of the screen by the screen not being filled, and the number of pixels for with one pixel will have to be added (+) or dropped (-) for the transformation from the base latitude to the radar latitude are shown. Also shown are the maximum tilts for the easternmost and westernmost NEXRAD's, the cotangent of the latitude, the cone parameter n, and the number of degrees over which the circle is mapped.

TANLAT	ETILT	WTILT	CTLAT	NCONE	DV360
10.000	4.9	-4.9	80.00	.174	62.5

LATITUDE .01 10.00 20.00 25.00 30.00 35.00 40.00 45.00 48.00 50.00 55.00 60.00 65.00 70.00 80.00 89.99
SIGMA 1.02 1.00 1.02 1.04 1.07 1.11 1.16 1.23 1.28 1.33 1.45 1.62 1.85 2.20 3.83******

	BASE LAT = 25.0				BASE LAT = 30.0				BASE LAT = 35.0			
RADAR LATITUDE	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)
25.0	1.00	460	26	****	.97	447	32	-36	.94	431	41	-16
30.0	1.03	473	19	35	1.00	460	26	****	.96	443	35	-27
35.0	1.07	491	10	15	1.04	478	17	26	1.00	460	26	****
40.0	1.12	516	-2	8	1.09	501	5	11	1.05	483	15	20
45.0	1.19	547	-18	5	1.16	532	-10	6	1.11	512	0	9
48.0	1.24	571	-29	4	1.21	555	-21	5	1.16	534	-11	6

TANLAT	ETILT	WTILT	CTLAT	NCONE	DV360
12.500	6.1	-6.1	77.50	.216	77.9

LATITUDE .01 10.00 20.00 25.00 30.00 35.00 40.00 45.00 48.00 50.00 55.00 60.00 65.00 70.00 80.00 89.99
SIGMA 1.02 1.00 1.01 1.02 1.05 1.09 1.13 1.20 1.24 1.28 1.39 1.54 1.75 2.06 3.48776.24

	BASE LAT = 25.0				BASE LAT = 30.0				BASE LAT = 35.0			
RADAR LATITUDE	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)
25.0	1.00	460	26	****	.98	449	31	-42	.94	434	39	-18
30.0	1.02	471	20	41	1.00	460	26	****	.97	445	34	-31
35.0	1.06	487	12	17	1.03	476	18	30	1.00	460	26	****
40.0	1.11	509	2	9	1.08	497	8	13	1.04	480	16	23
45.0	1.17	537	-13	6	1.14	524	-6	7	1.10	507	2	10
48.0	1.21	558	-23	5	1.18	545	-17	5	1.15	527	-8	7

TANLAT	ETILT	WTILT	CTLAT	NCONE	DV360
15.000	7.3	-7.3	75.00	.259	93.2

LATITUDE .01 10.00 20.00 25.00 30.00 35.00 40.00 45.00 48.00 50.00 55.00 60.00 65.00 70.00 80.00 89.99
SIGMA 1.03 1.00 1.00 1.02 1.04 1.07 1.11 1.16 1.21 1.24 1.34 1.47 1.66 1.93 3.17527.74

	BASE LAT = 25.0				BASE LAT = 30.0				BASE LAT = 35.0			
RADAR LATITUDE	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)
25.0	1.00	460	26	****	.98	451	31	-51	.95	438	37	-21
30.0	1.02	469	21	50	1.00	460	26	****	.97	447	33	-35
35.0	1.05	483	14	20	1.03	473	19	34	1.00	460	26	****
40.0	1.09	502	5	11	1.07	492	10	14	1.04	478	17	25
45.0	1.15	527	-8	7	1.12	517	-2	8	1.09	502	5	11
48.0	1.19	546	-17	5	1.16	536	-12	6	1.13	520	-4	8

Figure 3. Same as Fig. 2 except for tangents at 10.0^{o}, 12.5^{o}, and 15.0^{o}.

TANLAT	ETILT	WTILT	CTLAT	NCONE	DV360
17.500	8.5	-8.5	72.50	.301	108.3

LATITUDE	.01	10.00	20.00	25.00	30.00	35.00	40.00	45.00	48.00	50.00	55.00	60.00	65.00	70.00	80.00	89.99
SIGMA	1.05	1.01	1.00	1.01	1.02	1.05	1.09	1.14	1.17	1.20	1.29	1.41	1.57	1.82	2.90	361.09

	BASE LAT = 25.0				BASE LAT = 30.0				BASE LAT = 35.0			
RADAR LATITUDE	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)
25.0	1.00	460	26	****	.98	453	30	-64	.96	442	35	-25
30.0	1.02	467	22	63	1.00	460	26	****	.98	449	32	-41
35.0	1.04	479	17	24	1.02	471	20	40	1.00	460	26	****
40.0	1.08	495	8	13	1.06	488	12	17	1.03	476	18	29
45.0	1.13	518	-3	8	1.11	510	1	9	1.08	497	7	12
48.0	1.16	535	-12	6	1.14	527	-7	7	1.12	514	-1	9

TANLAT	ETILT	WTILT	CTLAT	NCONE	DV360
20.000	9.6	-9.7	70.00	.342	123.1

LATITUDE	.01	10.00	20.00	25.00	30.00	35.00	40.00	45.00	48.00	50.00	55.00	60.00	65.00	70.00	80.00	89.99
SIGMA	1.06	1.02	1.00	1.00	1.02	1.04	1.07	1.11	1.14	1.17	1.25	1.35	1.50	1.71	2.66	248.82

	BASE LAT = 25.0				BASE LAT = 30.0				BASE LAT = 35.0			
RADAR LATITUDE	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)
25.0	1.00	460	26	****	.99	455	29	-85	.97	445	33	-32
30.0	1.01	465	23	84	1.00	460	26	****	.98	451	31	-50
35.0	1.03	475	19	31	1.02	469	21	49	1.00	460	26	****
40.0	1.06	489	11	16	1.05	483	14	20	1.03	474	19	34
45.0	1.11	509	2	9	1.09	503	5	11	1.07	493	10	14
48.0	1.14	524	-6	7	1.13	518	-3	8	1.10	507	2	10

TANLAT	ETILT	WTILT	CTLAT	NCONE	DV360
22.500	10.8	-10.8	67.50	.383	137.8

LATITUDE	.01	10.00	20.00	25.00	30.00	35.00	40.00	45.00	48.00	50.00	55.00	60.00	65.00	70.00	80.00	89.99
SIGMA	1.08	1.02	1.00	1.00	1.01	1.03	1.05	1.09	1.12	1.14	1.21	1.30	1.43	1.62	2.44	172.79

	BASE LAT = 25.0				BASE LAT = 30.0				BASE LAT = 35.0			
RADAR LATITUDE	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)
25.0	1.00	460	26	****	.99	456	28	-129	.98	449	31	-42
30.0	1.01	464	24	128	1.00	460	26	****	.98	453	30	-63
35.0	1.02	471	20	41	1.02	467	22	62	1.00	460	26	****
40.0	1.05	483	15	20	1.04	479	16	24	1.03	472	20	40
45.0	1.09	500	6	11	1.08	496	8	13	1.06	488	12	16
48.0	1.12	513	-1	9	1.11	509	1	9	1.09	501	5	11

Figure 4. Same as Fig. 2 except for tangents at 17.5°, 20.0°, and 22.5°.

TANLAT	ETILT	WTILT	CTLAT	NCONE	OV360
25.000	11.9	-12.0	65.00	.423	152.1

LATITUDE	.01	10.00	20.00	25.00	30.00	35.00	40.00	45.00	48.00	50.00	55.00	60.00	65.00	70.00	80.00	89.99
SIGMA	1.10	1.03	1.00	1.00	1.00	1.02	1.04	1.07	1.09	1.11	1.17	1.26	1.37	1.54	2.26	121.01

	BASE LAT = 25.0				BASE LAT = 30.0				BASE LAT = 35.0			
RADAR LATITUDE	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)
25.0	1.00	460	26	****	1.00	458	27	-259	.98	453	30	-64
30.0	1.00	462	25	258	1.00	460	26	****	.99	455	29	-85
35.0	1.02	467	22	63	1.01	466	23	84	1.00	460	26	****
40.0	1.04	477	18	27	1.03	475	18	30	1.02	470	21	48
45.0	1.07	492	10	15	1.06	490	11	16	1.05	484	14	19
48.0	1.09	503	5	11	1.09	501	5	11	1.08	495	8	13

TANLAT	ETILT	WTILT	CTLAT	NCONE	OV360
27.500	13.0	-13.1	62.50	.462	166.2

LATITUDE	.01	10.00	20.00	25.00	30.00	35.00	40.00	45.00	48.00	50.00	55.00	60.00	65.00	70.00	80.00	89.99
SIGMA	1.12	1.05	1.01	1.00	1.00	1.01	1.03	1.05	1.07	1.09	1.14	1.22	1.32	1.47	2.09	85.51

	BASE LAT = 25.0				BASE LAT = 30.0				BASE LAT = 35.0			
RADAR LATITUDE	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)
25.0	1.00	460	26	****	1.00	460	26	****	.99	456	28	-128
30.0	1.00	460	26	****	1.00	460	26	****	.99	456	28	-128
35.0	1.01	464	24	127	1.01	464	24	127	1.00	460	26	****
40.0	1.02	471	20	41	1.02	471	20	41	1.02	468	22	61
45.0	1.05	483	14	20	1.05	483	14	20	1.04	480	16	24
48.0	1.07	493	9	14	1.07	493	9	14	1.06	489	11	16

TANLAT	ETILT	WTILT	CTLAT	NCONE	OV360
30.000	14.1	-14.1	60.00	.500	180.0

LATITUDE	.01	10.00	20.00	25.00	30.00	35.00	40.00	45.00	48.00	50.00	55.00	60.00	65.00	70.00	80.00	89.99
SIGMA	1.14	1.06	1.01	1.00	1.00	1.00	1.02	1.04	1.06	1.07	1.12	1.18	1.27	1.40	1.94	61.02

	BASE LAT = 25.0				BASE LAT = 30.0				BASE LAT = 35.0			
RADAR LATITUDE	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)
25.0	1.00	460	26	****	1.00	462	25	266	1.00	460	26	****
30.0	1.00	458	27	-267	1.00	460	26	****	1.00	458	27	-258
35.0	1.00	460	26	7774	1.00	462	25	257	1.00	460	26	****
40.0	1.01	466	23	82	1.02	467	22	63	1.01	466	23	83
45.0	1.03	475	18	30	1.04	477	17	27	1.03	475	18	30
48.0	1.05	484	14	19	1.06	485	13	18	1.05	484	14	20

Figure 5. Same as Fig. 2 except for tangents at 25.0°, 27.5°, and 30.0°.

TANLAT	TANLT1	TANLT2	ETILT	WTILT	CTLAT1	CTLAT2	NCONE	OV360
10.000	10.010	9.990	4.9	-4.9	79.99	80.01	.174	62.5

LATITUDE	.01	10.00	20.00	25.00	30.00	35.00	40.00	45.00	48.00	50.00	55.00	60.00	65.00	70.00	80.00	89.99
SIGMA	1.02	1.00	1.02	1.04	1.07	1.11	1.16	1.23	1.28	1.33	1.45	1.62	1.85	2.20	3.83	******

	BASE LAT = 25.0				BASE LAT = 30.0				BASE LAT = 35.0			
RADAR LATITUDE	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)
25.0	1.00	460	26	****	.97	447	32	-36	.94	431	41	-16
30.0	1.03	473	19	35	1.00	460	26	****	.96	443	35	-27
35.0	1.07	491	10	15	1.04	478	17	26	1.00	460	26	****
40.0	1.12	516	-2	8	1.09	501	5	11	1.05	483	15	20
45.0	1.19	547	-18	5	1.16	532	-10	6	1.11	512	0	9
48.0	1.24	571	-29	4	1.21	555	-21	5	1.16	534	-11	6

TANLAT	TANLT1	TANLT2	ETILT	WTILT	CTLAT1	CTLAT2	NCONE	OV360
10.000	15.000	5.000	4.9	-4.9	75.00	85.00	.174	62.6

LATITUDE	.01	10.00	20.00	25.00	30.00	35.00	40.00	45.00	48.00	50.00	55.00	60.00	65.00	70.00	80.00	89.99
SIGMA	1.01	1.00	1.01	1.03	1.06	1.10	1.16	1.23	1.28	1.32	1.44	1.61	1.84	2.19	3.81	******

	BASE LAT = 25.0				BASE LAT = 30.0				BASE LAT = 35.0			
RADAR LATITUDE	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)
25.0	1.00	460	26	****	.97	447	32	-36	.94	431	41	-16
30.0	1.03	473	19	35	1.00	460	26	****	.96	443	34	-27
35.0	1.07	491	10	15	1.04	478	17	26	1.00	460	26	****
40.0	1.12	515	-2	8	1.09	501	5	11	1.05	483	15	20
45.0	1.19	547	-18	5	1.16	532	-10	6	1.11	512	0	9
48.0	1.24	571	-29	4	1.21	555	-21	5	1.16	534	-11	6

TANLAT	TANLT1	TANLT2	ETILT	WTILT	CTLAT1	CTLAT2	NCONE	OV360
10.000	20.000	.010	4.9	-4.9	70.00	89.99	.175	62.9

LATITUDE	.01	10.00	20.00	25.00	30.00	35.00	40.00	45.00	48.00	50.00	55.00	60.00	65.00	70.00	80.00	89.99
SIGMA	1.00	.98	1.00	1.02	1.05	1.09	1.14	1.21	1.26	1.30	1.43	1.59	1.82	2.16	3.76	******

	BASE LAT = 25.0				BASE LAT = 30.0				BASE LAT = 35.0			
RADAR LATITUDE	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)
25.0	1.00	460	26	****	.97	447	32	-36	.94	431	41	-16
30.0	1.03	473	19	35	1.00	460	26	****	.96	443	34	-27
35.0	1.07	491	10	15	1.04	478	17	26	1.00	460	26	****
40.0	1.12	515	-2	8	1.09	501	6	11	1.05	483	15	20
45.0	1.19	547	-17	5	1.16	532	-10	6	1.11	512	0	9
48.0	1.24	570	-29	4	1.21	554	-21	5	1.16	534	-11	6

Figure 6. Same as Fig. 2 except for the secant Lambert with intersections at 10.01° and 9.99° (essentially a Lambert tangent at 10.0°), 15.0° and 5.0°, and 20.0° and $.01^{\circ}$. The latitudes (TANLT1 and TANLT2) and colatitudes (CTLAT1 and CTLAT2) are given for the northern and southern intersections, respectively.

TANLAT	TANLT1	TANLT2	ETILT	WTILT	CTLAT1	CTLAT2	NCONE	OV360
22.500	22.510	22.490	10.8	-10.8	67.49	67.51	.383	137.7

LATITUDE	.01	10.00	20.00	25.00	30.00	35.00	40.00	45.00	48.00	50.00	55.00	60.00	65.00	70.00	80.00	89.99
SIGMA	1.08	1.02	1.00	1.00	1.01	1.03	1.05	1.09	1.12	1.14	1.21	1.30	1.43	1.62	2.44	172.95

	BASE LAT = 25.0				BASE LAT = 30.0				BASE LAT = 35.0			
RADAR LATITUDE	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)
25.0	1.00	460	26	****	.99	456	28	-129	.98	449	31	-42
30.0	1.01	464	24	128	1.00	460	26	****	.98	453	30	-63
35.0	1.02	471	20	41	1.02	467	22	62	1.00	460	26	****
40.0	1.05	483	15	20	1.04	479	16	24	1.03	472	20	40
45.0	1.09	500	6	11	1.08	496	8	13	1.06	488	12	16
48.0	1.12	513	-1	9	1.11	509	1	9	1.09	501	5	11

TANLAT	TANLT1	TANLT2	ETILT	WTILT	CTLAT1	CTLAT2	NCONE	OV360
22.500	32.500	12.500	10.8	-10.9	57.50	77.50	.385	138.5

LATITUDE	.01	10.00	20.00	25.00	30.00	35.00	40.00	45.00	48.00	50.00	55.00	60.00	65.00	70.00	80.00	89.99
SIGMA	1.06	1.01	.99	.99	.99	1.01	1.03	1.07	1.10	1.12	1.19	1.28	1.41	1.59	2.40	167.20

	BASE LAT = 25.0				BASE LAT = 30.0				BASE LAT = 35.0			
RADAR LATITUDE	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)
25.0	1.00	460	26	****	.99	457	28	-132	.98	449	31	-43
30.0	1.01	464	24	131	1.00	460	26	****	.98	453	30	-64
35.0	1.02	471	21	42	1.02	467	22	63	1.00	460	26	****
40.0	1.05	483	15	20	1.04	479	16	24	1.02	471	20	40
45.0	1.09	500	6	12	1.08	496	8	13	1.06	488	12	16
48.0	1.11	513	0	9	1.11	509	2	9	1.09	501	6	11

TANLAT	TANLT1	TANLT2	ETILT	WTILT	CTLAT1	CTLAT2	NCONE	OV360
22.500	42.500	2.500	11.0	-11.1	47.50	87.50	.391	140.7

LATITUDE	.01	10.00	20.00	25.00	30.00	35.00	40.00	45.00	48.00	50.00	55.00	60.00	65.00	70.00	80.00	89.99
SIGMA	1.02	.96	.94	.94	.95	.96	.98	1.02	1.04	1.07	1.13	1.21	1.33	1.51	2.26	150.87

	BASE LAT = 25.0				BASE LAT = 30.0				BASE LAT = 35.0			
RADAR LATITUDE	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)	SCALE RATIO	CIRCLE DIA	SCREEN WASTE	ADD (+) DROP(-)
25.0	1.00	460	26	****	.99	457	28	-144	.98	450	31	-46
30.0	1.01	463	24	143	1.00	460	26	****	.98	453	29	-66
35.0	1.02	470	21	45	1.02	467	22	65	1.00	460	26	****
40.0	1.05	482	15	21	1.04	478	17	25	1.02	471	20	41
45.0	1.08	498	7	12	1.08	495	9	13	1.06	487	12	17
48.0	1.11	511	0	9	1.10	508	2	10	1.09	500	6	12

Figure 7. Same as Fig. 6 except for intersections at 22.51° and 22.49° (essentially a Lambert tangent at 22.5°), 32.5° and 12.5°, and 42.5° and 2.5°.

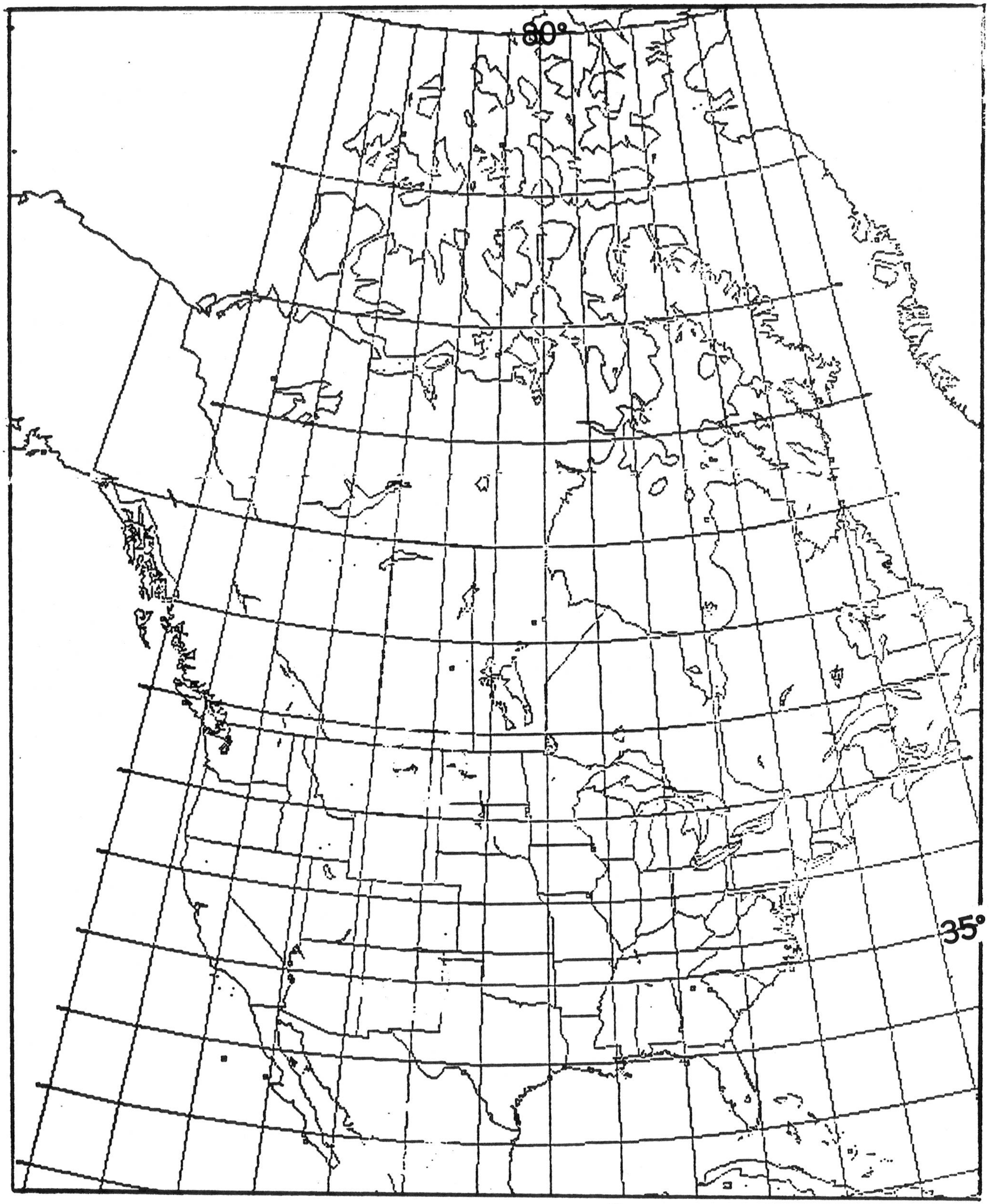


Figure 8. A Lambert conformal map tangent at 25° N. The northern extent is about 80°.

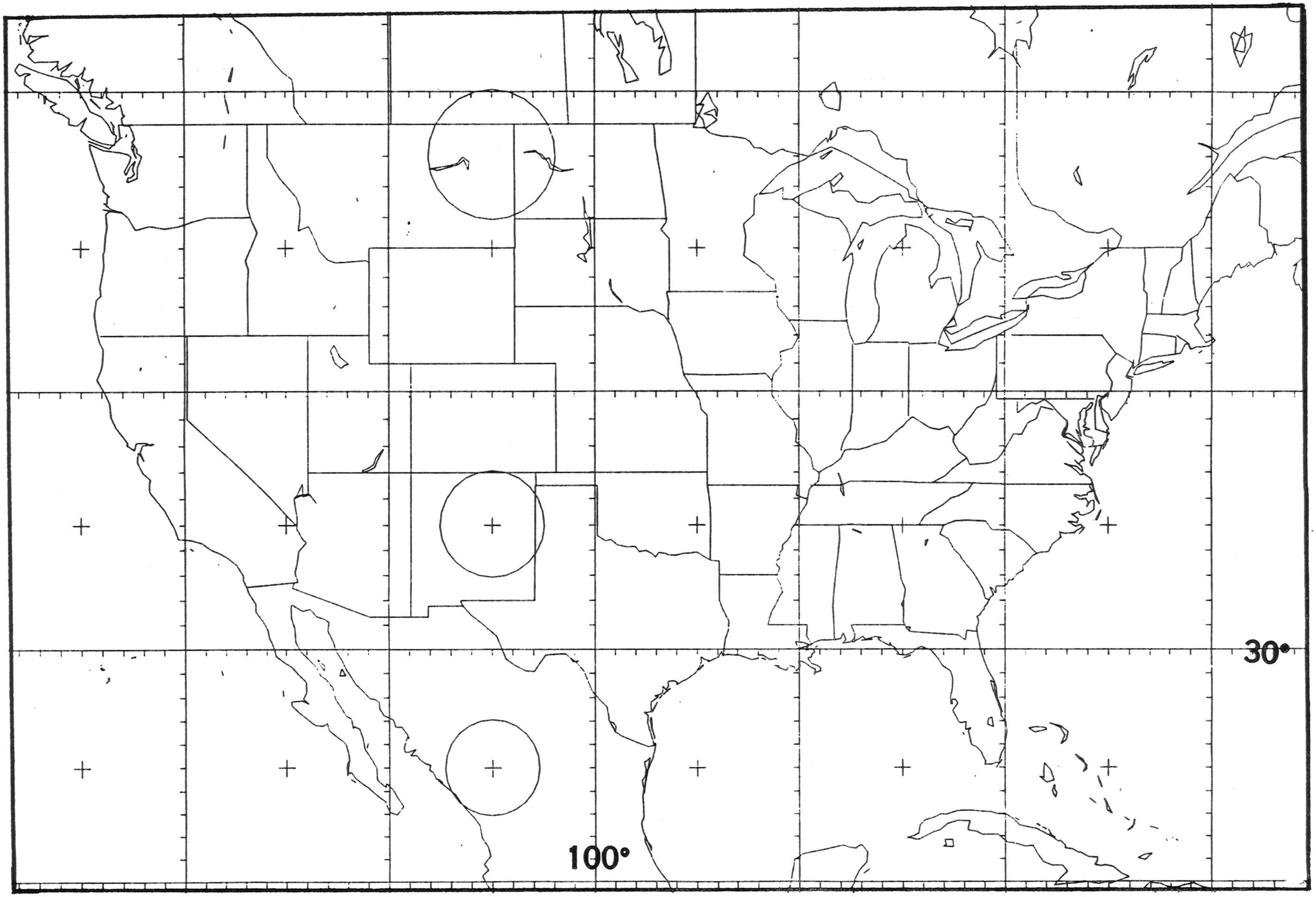


Figure 9. A Mercator map covering much of the area that would be transmitted from NMC to support the Regional Areas. Circles represent radar umbrellas of 460-km diameter at 25°, 35°, and 48° N.

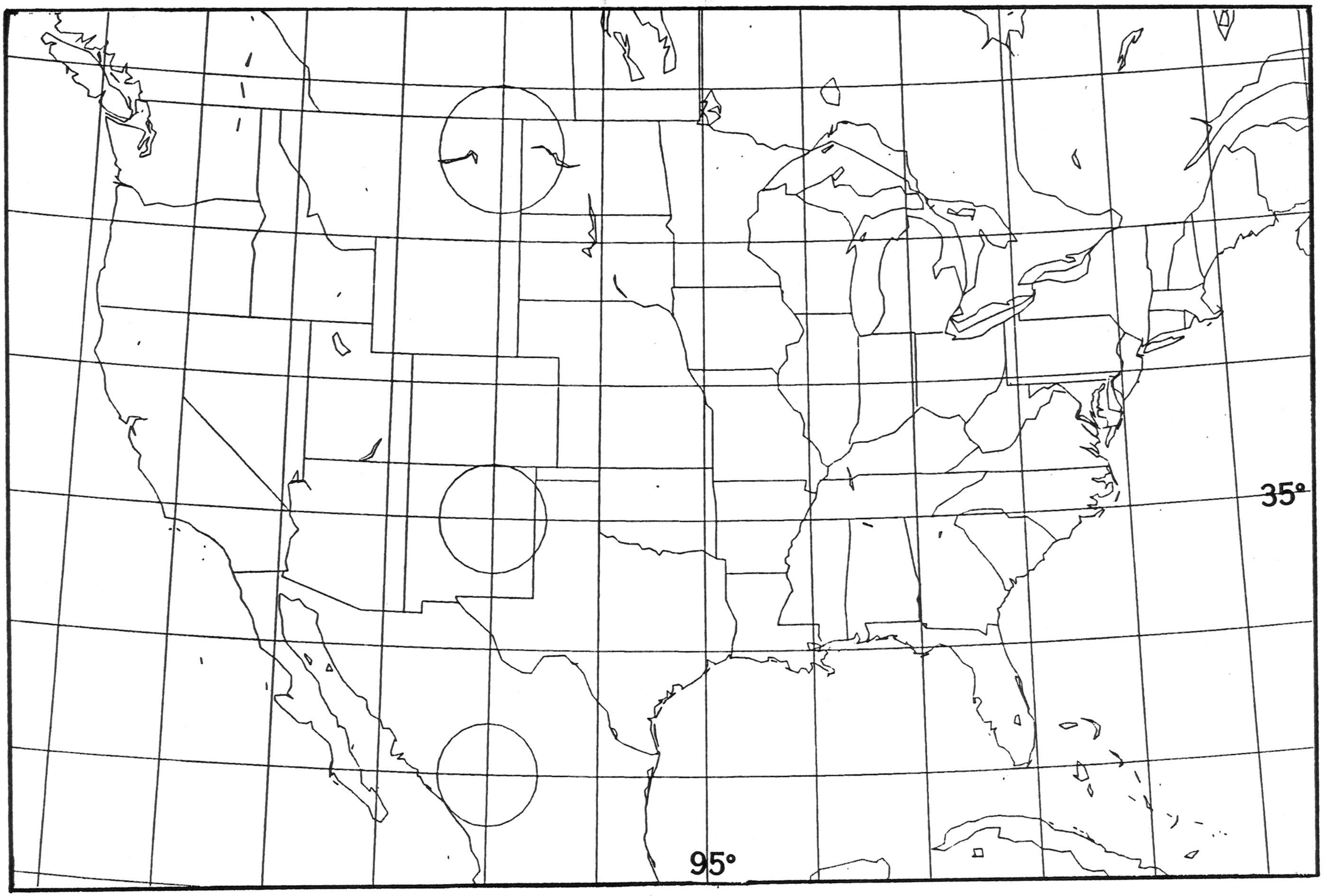


Figure 10. Same as Fig. 9, except a Lambert conformal tangent at 10^{o} N.

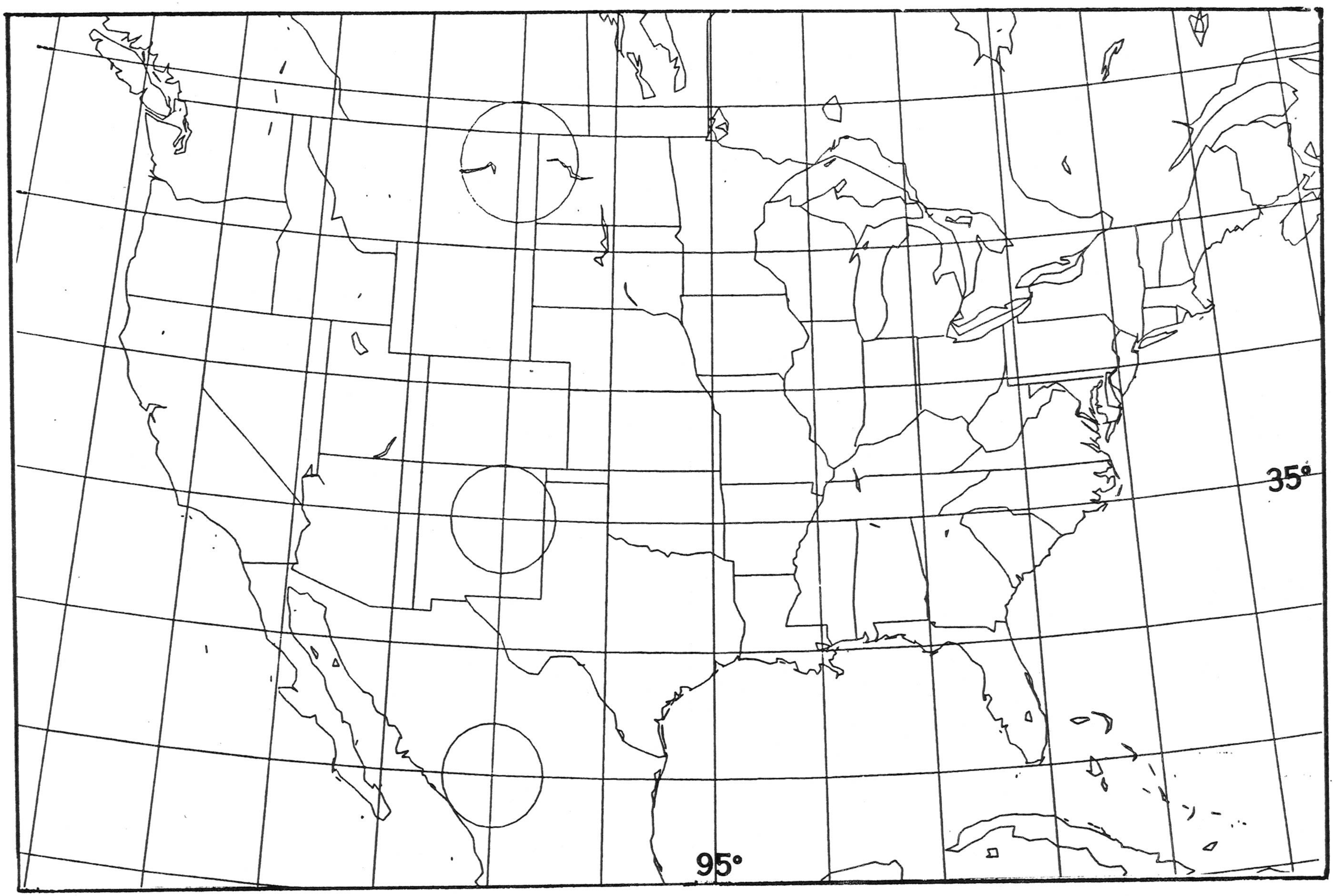


Figure 11. Same as Fig. 9, except a Lambert conformal tangent at 17.5° N.

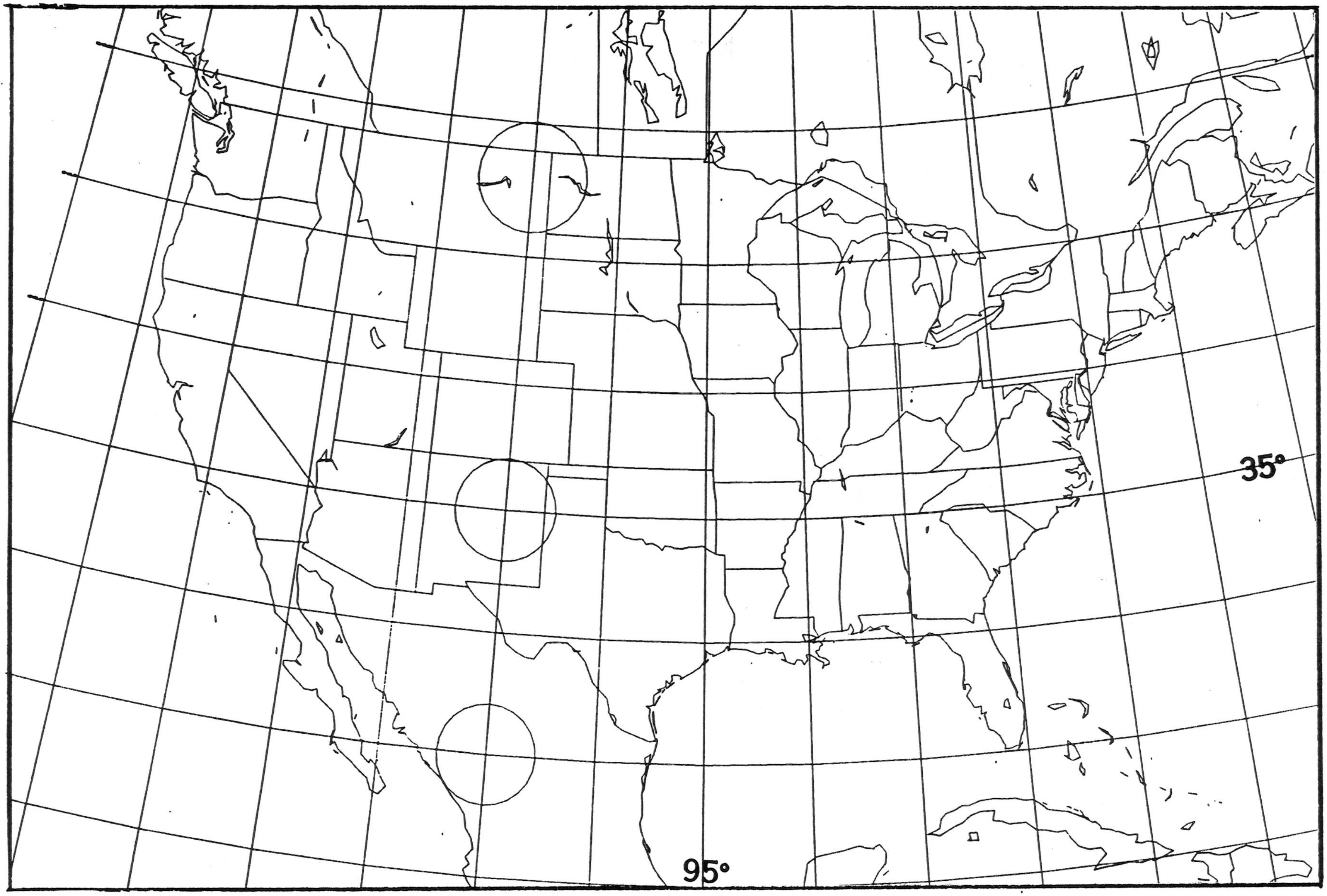


Figure 12. Same as Fig. 9, except a Lambert conformal tangent at 25^{o} N.

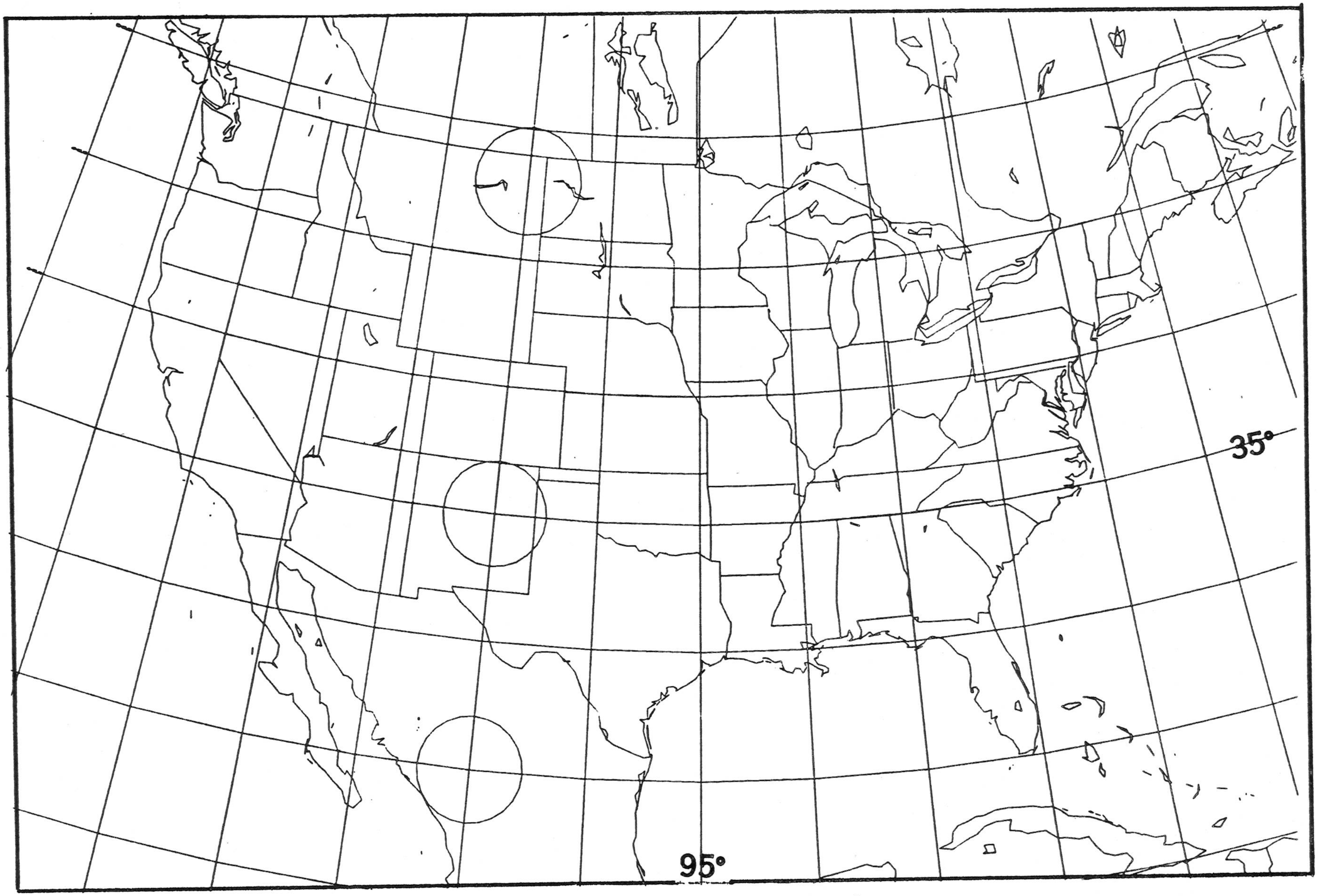


Figure 13. Same as Fig. 9, except a Lambert conformal tangent at 36.8^{o} N.

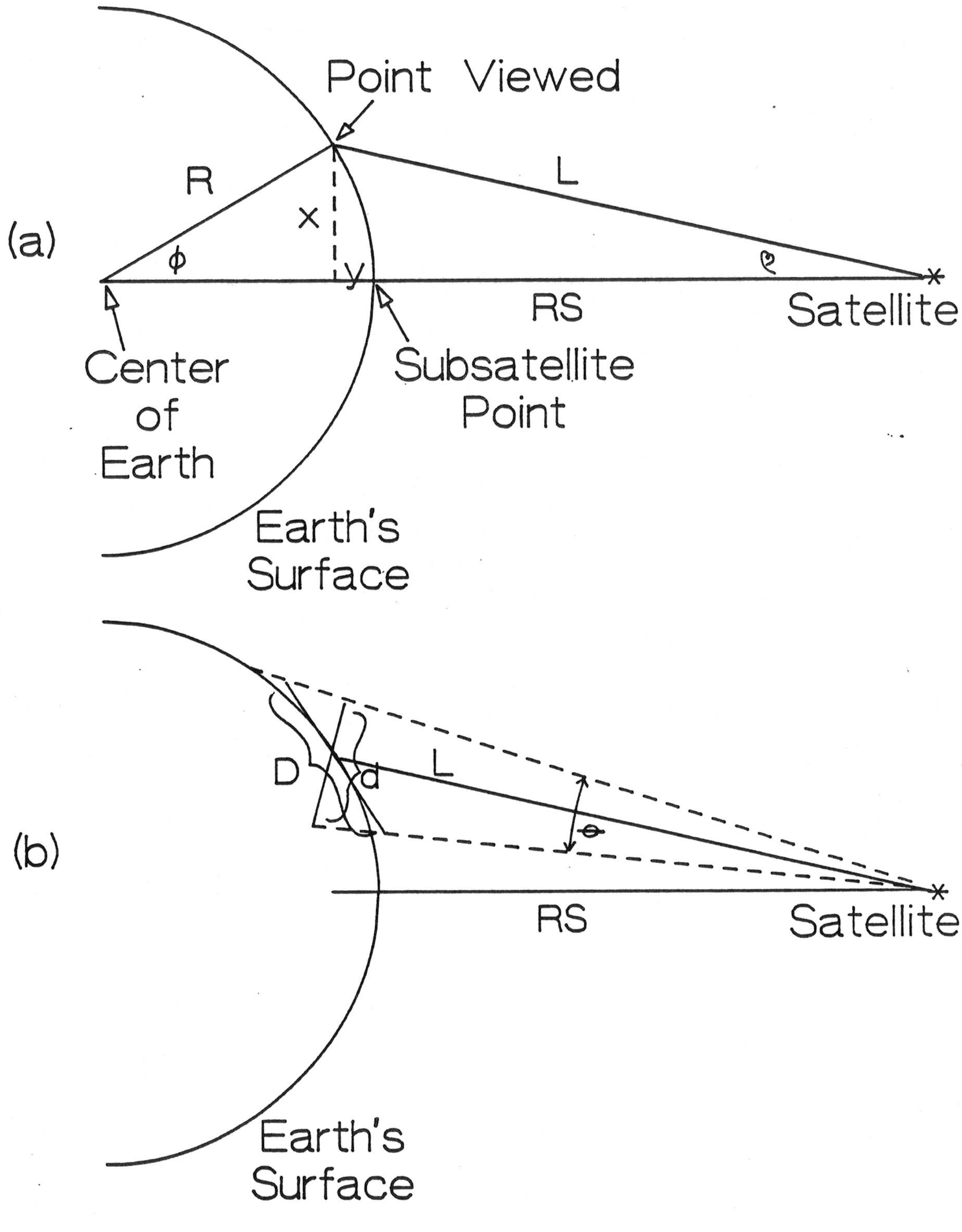


Figure 14. Geometric relationships used for computing the dimensions d and D of the spot viewed by GOES. (b) is an expanded view of a portion of (a).

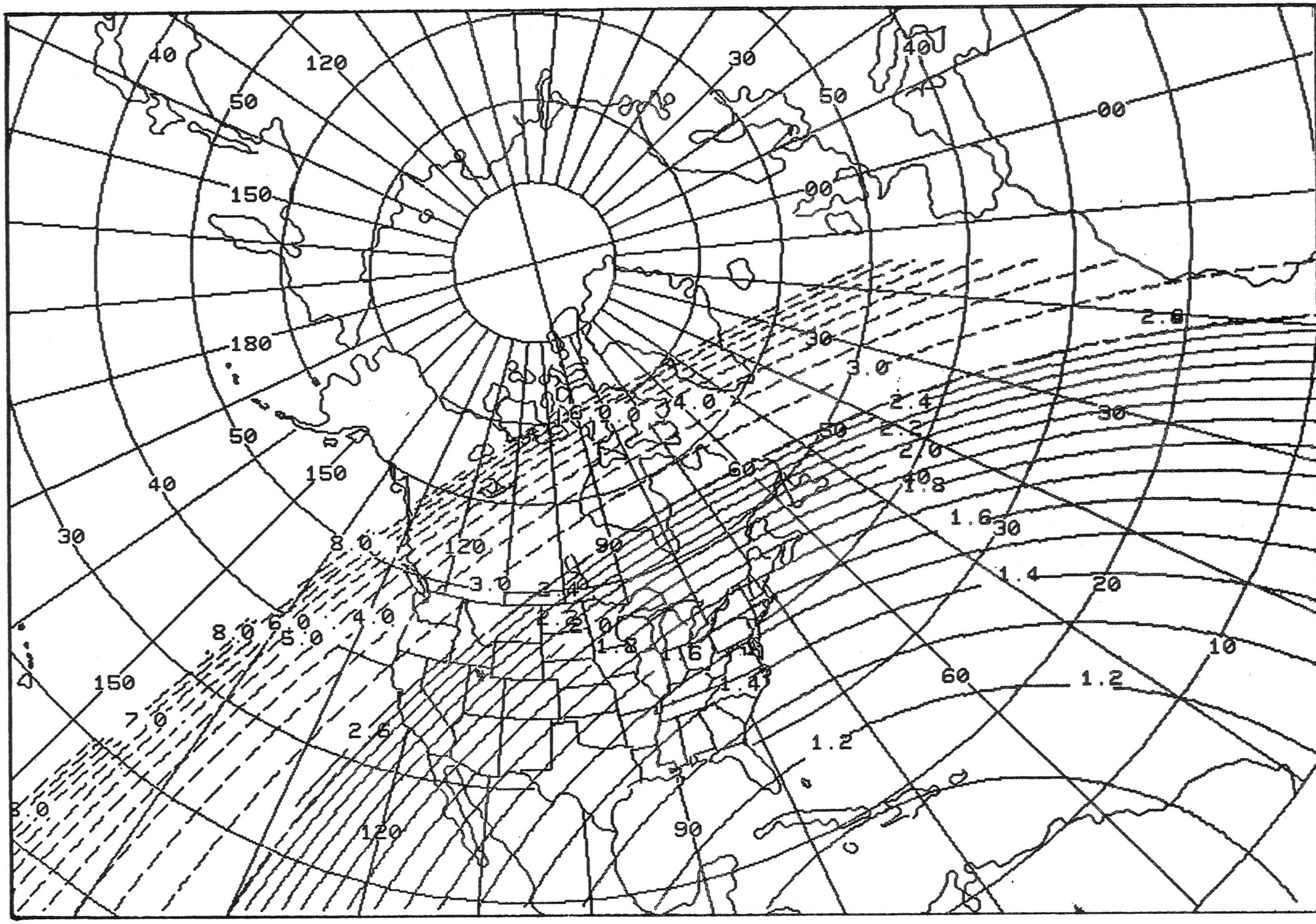


Figure 15. The maximum dimension D of the rectangle viewed by GOES EAST located at 75° W. Contouring is at intervals of 0.1 km (solid lines), then at intervals of 0.5 km (dashed lines).

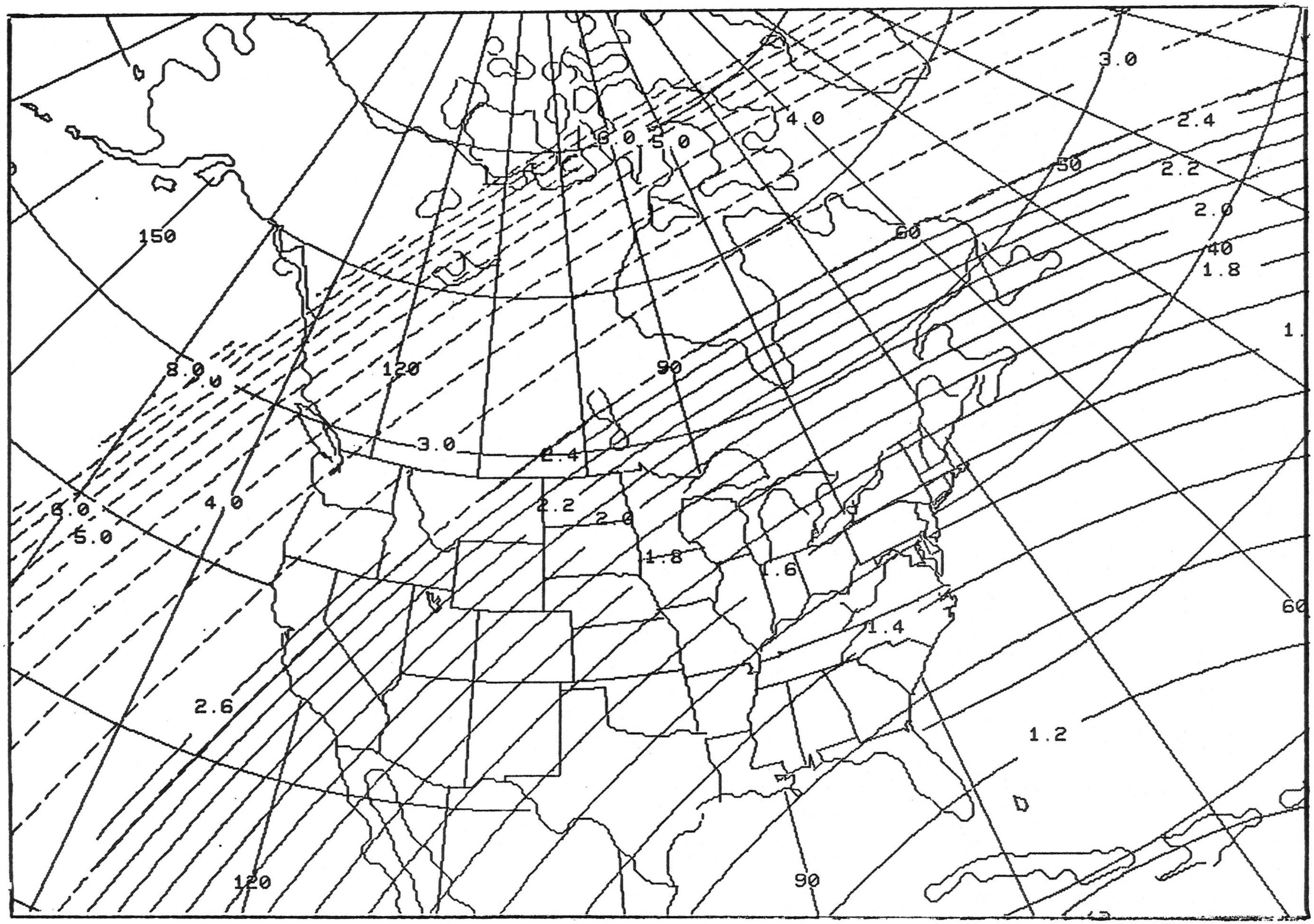


Figure 16. Same as Fig. 15 except a portion enlarged for easier reading.

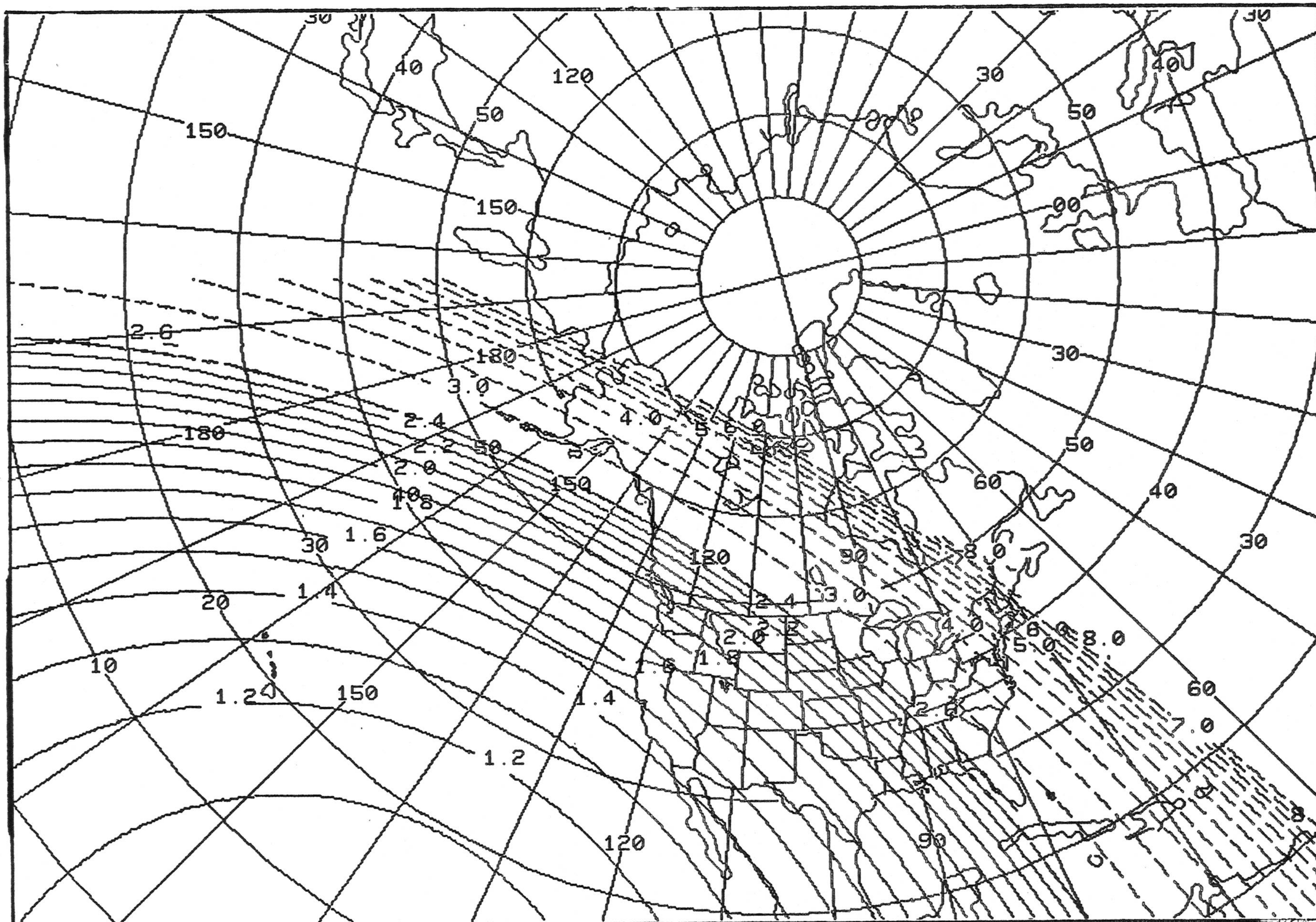


Figure 17. Same as Fig. 15 except for GOES WEST located at 135° W.

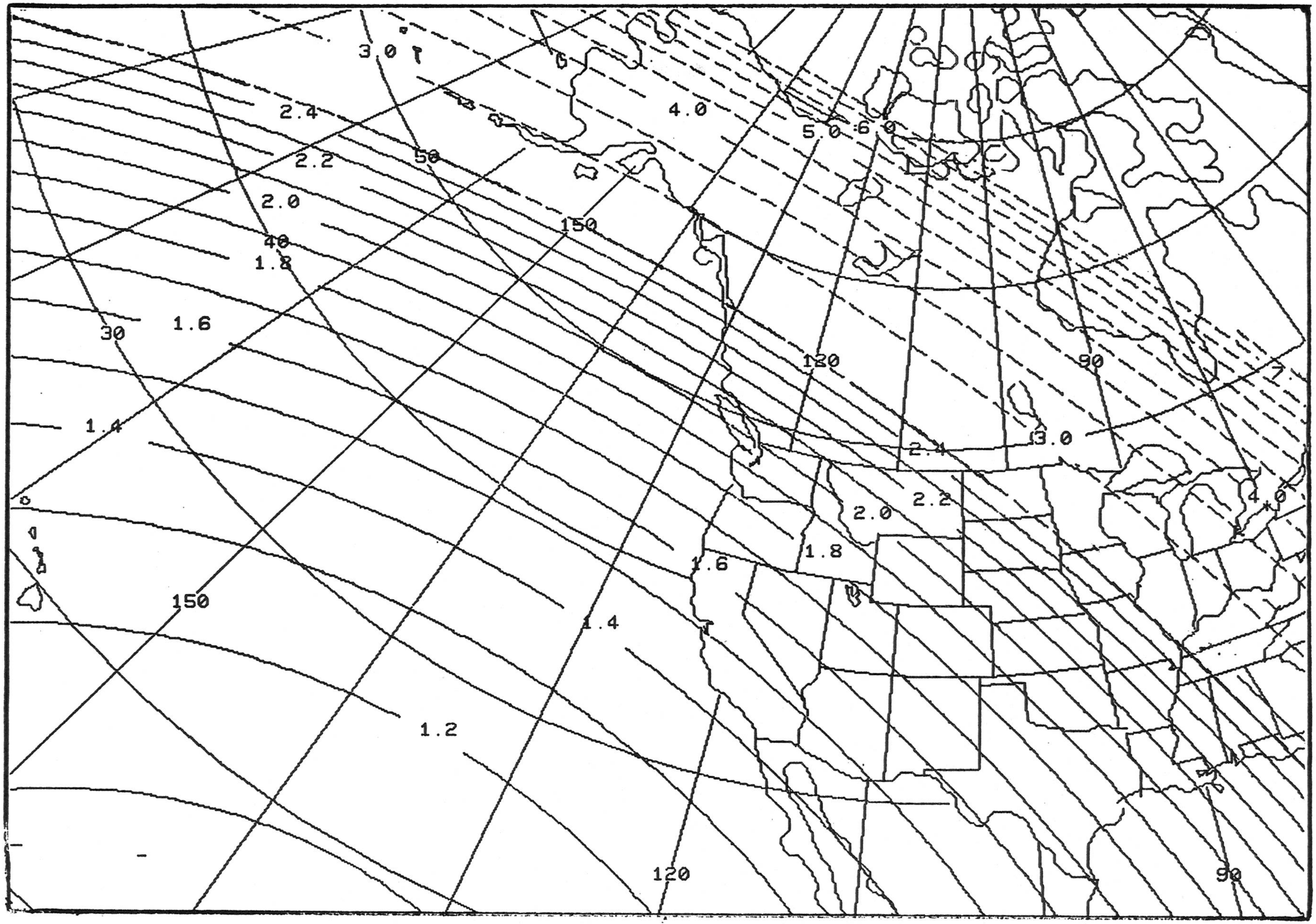


Figure 18. Same as Fig 17, except a portion enlarged for easier reading.

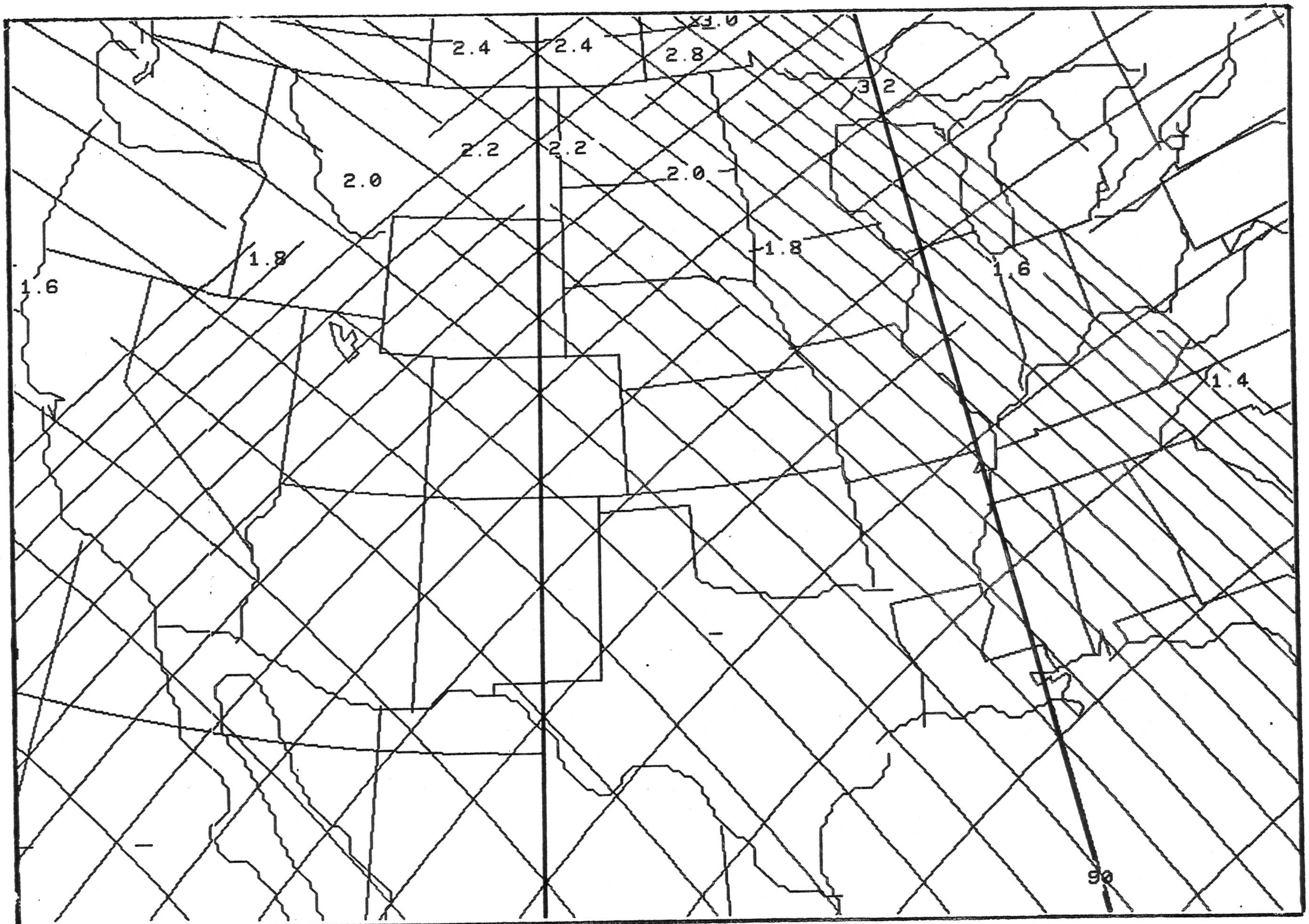


Figure 19. The maximum dimension D of the rectangle viewed by GOES EAST and GOES WEST. Contouring is at intervals of 0.1 km. Longitudes 90° and 105° W are highlighted. Note that the values are equal at 105° W for both satellites.

Table 1. Summary of possible combinations of latitude of tangency and the latitude at which the scale is 1 km/pixel. Negative Values in column six (seven) means that pixels are added (dropped) instead of dropped (added). Values in parentheses in the last column mean that the screen is not filled at any latitude and could accommodate that many more pixels at each edge. The scale at a particular latitude is computed from the appropriate 1/R(T,B) = 460 km/(CIRCLE DIAMETER) in pixels. The tangent at 36.8° (37.6°) corresponds to secant intersections at 25° and 48° (30° and 45°).

Latitude of Tangency (Deg)	Latitude at Which Scale is 1 km/Pixel (Deg)	Maximum Tilt (Deg)	Scale at Latitude 25° (km/Pixel)	Scale at Latitude 48° (km/Pixel)	Pixels Dropped at 25° Lat (1 in n)	Pixels Added at 48° Lat (1 in n)	Maximum Pixels Wasted at each edge	Maximum Pixels Lost at each edge
0.0	35	0.0	1.11	0.82	10	4	48	26
0.0	30	0.0	1.05	0.77	23	3	36	42
0.0	25	0.0	1.00	0.74	—	3	26	56
10.0	35	4.9	1.07	0.86	16	6	41	11
10.0	30	4.9	1.03	0.83	36	5	32	21
10.0	25	4.9	1.00	0.81	—	4	26	29
15.0	30	7.3	1.02	0.86	51	6	31	12
17.5	35	8.5	1.04	0.89	25	9	35	1
17.5	30	8.5	1.02	0.87	64	7	30	7
17.5	25	8.5	1.00	0.86	—	6	26	12
20.0	25	9.7	1.00	0.88	—	7	26	6
22.5	35	10.8	1.02	0.92	42	11	31	(5)
25.0	35	12.0	1.02	0.93	64	13	30	(8)
25.0	25	12.0	1.00	0.91	—	11	26	(5)
30.0	35	14.1	1.00	0.95	—	20	27	(14)
35.0	35	16.2	0.99	0.97	-67	36	26	(20)
36.8	35	16.9	0.98	0.98	-50	50	26	(21)
36.8	30	16.9	0.99	0.99	-74	74	27	(23)
36.8	25	16.9	1.00	1.00	—	—	31	(26)
37.6	35	17.3	0.98	0.98	-45	61	26	(21)

Table 2. Approximate percent of area of the 48 states with less than the indicated tilt angles for a Lambert projection tangent at 17.5° with a maximum tilt of 8.5 degrees.

Tilt (deg)	Area (Percent)
1	15
2	30
3	43
4	56
5	69
6	80
7	88
8	95

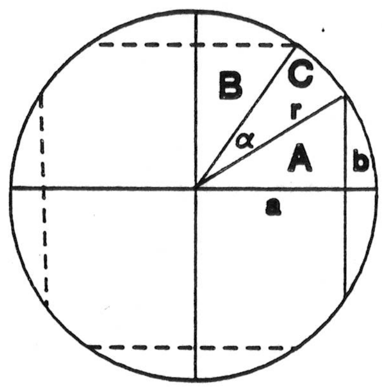
Table 3. Percent of circle area lost as a function of pixels lost on the radius.

RADIUS OF CIRCLE	PIXELS LOST AT EACH EDGE	PERCENT LOST ON RADIUS	PERCENT AREA LOST
256	0	.0	.0
261	5	1.9	.6
266	10	3.8	1.7
271	15	5.5	3.1
276	20	7.2	4.6
281	25	8.9	6.3
286	30	10.5	8.0
291	35	12.0	9.8
296	40	13.5	11.7
301	45	15.0	13.6
306	50	16.3	15.5
311	55	17.7	17.4
316	60	19.0	19.3
321	65	20.2	21.2
326	70	21.5	23.1
331	75	22.7	25.0
336	80	23.8	26.9
341	85	24.9	28.7
346	90	26.0	30.6
351	95	27.1	32.4
356	100	28.1	34.2
361	105	29.1	36.0

APPENDIX I

Area of Circle Lost as it is Cropped by a Square of Dimensions Less than the Circle Diameter

Given that a circle cannot be inscribed in a particular square, what area of the circle is lost by cropping equal portions at each edge? Consider the first quadrant. The area of the circle within the square is composed of portions A, B, and C, where A and C are equal (see the diagram below).



The areas A, B, and C are

$$A = B = ab/2 = a(r^2 - a^2)^{1/2}/2$$

and

$$C = r^2 \alpha/2,$$

where

$$\alpha = \pi/2 - 2 \text{ arc } \cos(a/r).$$

Therefore, the total area of the circle not cropped by the square is

$$\text{Area not cropped} = 4a(r^2 - a^2)^{1/2} + \pi r^2 - 4r^2 \text{ arc } \cos(a/r).$$

Also, for a square where a is half the side length, the fraction of area lost when $r \geq a$ is

$$\text{Percent of area lost} = 4 \text{ arc } \cos(a/r)/\pi - 4a(r^2 - a^2)^{1/2}/\pi r^2.$$

These equations hold until $\alpha = 0^o$ which occurs when the circle circumscribes the square.

Suppose a viewing screen 512 pixels square and radar data existing over a circle 512 pixels or larger in diameter. Table 3 shows the percent of area lost as a function of the number of pixels lost at each edge and the corresponding percent of radius lost.

APPENDIX II

Actual Dimensions of GOES IJK/LM Nominal 1-km Data

The spot on the earth viewed by the GOES IJK/LM in the visible wavelength is very close to a square 1 km on a side at the subsatellite point, according to information furnished by NESDIS. This spot becomes larger at other points on the earth due, primarily, to two effects. First, since the aperture of the imager is constant, the area seen on the earth varies with distance to the earth's surface, which increases as the distance on the earth from the subsatellite point increases. This effect accounts for an increase in each dimension of a viewed spot on a plane perpendicular to the "line of sight"--a line drawn from the point on the earth to the satellite. This effect is relatively minor, as we shall see, but does change each dimension of the square (call it "d") to about 1.1 km in the central U.S.

The other effect is due to the surface of the earth not being perpendicular to the line of sight. At the longitude of the satellite, this effect is a function of latitude. The spot viewed at latitude ϕ (at the longitude of the satellite) is a rectangle with dimensions d and D (neglecting the curvature of the earth over the area of the spot), D being in the direction of the satellite. The dimensions D and d can be computed from latitude ϕ and known constants.

Consider Fig. 14. The known values are θ, the effective viewing angle of the imager = 28 μradians; R, the radius of the earth, here assumed to be constant at the equatorial radius of 6378 km; and RS, the height of the satellite above the subsatellite point = 35792 km. The following can be computed:

$$X = R \operatorname{Sin} \phi ,$$

$$Y = R(1 - \operatorname{Cos} \phi),$$

$$L = \left[(RS + Y)^2 + X^2\right]^{1/2} ,$$

$$\varrho = \operatorname{Cos}^{-1} \left[\frac{RS + Y}{L}\right] ,$$

$$d = 2\, L \operatorname{Tan}(\theta/2), \text{ and}$$

$$D = d/\operatorname{Cos}(\phi + \varrho).$$

The above relationships hold for any point on the earth that can be viewed by the satellite, provided the latitude ϕ is replaced in the equations with the angle ϕ', which is the angle subtended by an arc on the earth between the subsatellite point and the point viewed. Such an angle can be computed between two points having coordinates Lat1, Long1 and Lat2, Long2, respectively, by

$$\phi' = \operatorname{Cos}^{-1}\left[(\operatorname{Sin} \text{Lat1})(\operatorname{Sin} \text{Lat2}) + (\operatorname{Cos} \text{Lat1})(\operatorname{Cos} \text{Lat2})(\operatorname{Cos} \left|\text{Long1} - \text{Long2}\right|)\right]$$

Since the subsatellite point is at Lat1 = 0, and the latitude of the point viewed is ϕ,

$$\phi' = \operatorname{Cos}^{-1}\left[(\operatorname{Cos} \phi)(\operatorname{Cos} \left|\text{Long1} - \text{Long2}\right|)\right]$$

As a sample calculation, consider $\phi = 35.7^{\circ}$ N, Long1 = 105° W, and Long2 = 75° W (the longitude planned for GOES EAST). Then:

$X = 4535$ km,

$Y = 1893$ km,

$L = 37957$ km,

$\varrho = 6.86$ degrees,

$d = 1.06$ km,

$\phi' = 45.31$ degrees, and

$D = 1.73$ km.

The rectangle viewed is 1.06 by 1.73 km for a point in the central U.S. along 105° W. Since this longitude is halfway between the longitudes planned for GOES EAST and GOES WEST (135°), this is the best resolution that will be available for that point.

The dimension d is usually not greater than 1.1 km for areas with usable data; at latitude 61.2° N and longitude 105° W, $d = 1.12$ km, at which point $D = 3.99$ km.

Figs. 15 and 16 depict the dimension D for GOES EAST at 75° W on a polar stereographic map. Contours are every 0.1 km up to $D = 2.5$ km, then every 0.5 km up to 8.5 km. Fig. 15 shows much of the area that can be viewed by the satellite in the northern hemisphere; Fig. 16 is an expanded view of a portion of the area for easier reading.

Figs. 17 and 18 show for GOES WEST, planned for 135° W, the same information as Fig. 15 and 16 for GOES EAST. Fig. 19 shows lower values of D for both GOES EAST and WEST.

APPENDIX III

Map Factors

The image scale σ on a map projection is defined as the distance on the image surface (I) (at latitude ϕ) divided by the earth distance (E) at the point of intersection (or tangency) of the image surface with the earth:

$$\sigma = I/E.$$

So, if one knows I (for instance, a grid distance on the map), E can be computed as

$$E = I/\sigma.$$

On the other hand, the grid distance may be known at some other latitude than that of intersection. One can define a map factor m_s that pertains to the distance, or scale, being known at latitude s:

$$m_s = \sigma / \sigma_s.$$

where σ_s is σ at latitude s. m_s, in effect, serves the same purpose as σ where the scale is defined at latitude s rather than that of intersection. So, to get an earth distance, one would compute

$$E = I/m_s = I \sigma_s / \sigma.$$

This definition of map factor is consistent with much of the meteorological literature (e.g., Gerrity, 1973), although it has sometimes been defined to include the "shrinkage" from the image plane the the actual map used (e.g., Hoke, 1981, p. 3).

A. Polar Stereographic Projection

NMC uses the polar stereographic map projection with the scale defined at 60° N. Therefore, the map factor

$$m_{60} = \frac{\sigma}{\sigma_{60}} = \frac{2}{1 + \sin \phi} \times \frac{1 + \sin 60^\circ}{2} = \frac{1.866}{1 + \sin \phi} .$$

An earth distance can be computed by dividing the distance on the image plane by m_{60}. Note that this M_{60} is the same as σ when the image plane intersects the earth at 60° N latitude rather than being tangent at the pole; the secant latitude is relevant only in stating where the scale is defined. Evaluation of m_{60} multiplied by 2/1.866 gives the values for σ in Fig. 2 for TANLAT = 89.999. The reason for this multiplicative factor is that the values in Fig. 2 are based on the scale being defined at the pole.

B. Mercator Projection

Suppose that the Mercator projection were used with scale defined at 60° N. The map factor would be

$$m_{60} = \frac{\sigma}{\sigma_{60}} = \frac{1}{\cos\phi} \times \frac{\cos 60^{\circ}}{1} = \frac{0.5}{\cos\phi} .$$

Similarly, if the scale were defined at 35° N, the map factor would be

$$m_{35} = \frac{1}{\cos\phi} \times \frac{\cos 35^{\circ}}{1} = \frac{0.819}{\cos\phi} .$$

m_{35} is the same as σ when the cylinder onto which the earth's surface is projected cuts the earth at 35° N and 35° S. The secant latitude is relevant only in stating where the scale is defined. Evaluation of m_{60} multiplied by 1/0.5 gives the values in Fig. 2 for TANLAT = .001. As for the polar stereographic, the multiplier just compensates for where the scale is defined.

C. Lambert Conformal Projection

Suppose the Lambert conformal projection tangent at 25° were used with scale defined at 60° N. The map factor would be:

$$m_{60} = \frac{\sigma}{\sigma_{60}} = \frac{\frac{\sin 25^{\circ}}{\sin\psi}\left[\frac{\tan \psi/2}{\tan 25^{\circ}}\right]^{\cos 65^{\circ}}}{\frac{\sin 25^{\circ}}{\sin 30^{\circ}}\left[\frac{\tan 30^{\circ}/2}{\tan 25^{\circ}}\right]^{\cos 65^{\circ}}} = \frac{0.872\ (\tan \psi/2)^{0.423}}{\sin\psi}$$

For a scale defined at 35°, the map factor is

$$m_{35} = \frac{1.480\ (\tan \psi/2)^{0.906}}{\sin\psi}$$

Evaluation of m_{60} multiplied by 1.26 gives the values for σ in Fig. 5 for TANLAT = 25. Again, the multiplier compensates for where the scale is defined.

APPENDIX IV

Product Sizes

A. National Area

Fig. 1 of Appendix K (op. cit.) shows a map of the area over which NMC data would be transmitted to support all WFOs' National Areas. If the southern and northern extents of the area are 20° and 87° N, respectively, the distance along the 105° meridian of the image plane (IP) for a polar stereographic projection secant at 60° is (see Appendix III)

$$IP = \int_{20^\circ}^{87^\circ} \frac{1.866}{1 + \sin\phi}\, r\, d\phi = -1.866r \left[\tan(45^\circ - \frac{\phi}{2}) \right]_{20^\circ}^{87^\circ}$$

$$= 8587 \text{ km}$$

for a mean earth radius of 6370 km. The 1-Bedient scale at 60° N is 381 km, so the number of grid spaces on the plane is about 22.5, giving the number of gridpoints as, say, 23.

The east-west extent of the area to be transmitted at the lower boundary is approximately 7.5° N 142° W to 7.5° N 68° W. At 7.5° N, the map factor is

$$m = \frac{1.866}{1 + \sin\phi} = \frac{1.866}{1.131} = 1.65.$$

This means that along latitude circle 7.5°, the 1-Bedient distance on the earth is 381/1.65 = 231 km. The distance around the earth at latitude 7.5° is $2\pi r \cos 7.5^\circ = 39681$ km, which is 39681/231 = 172 grid distances. So, latitude 7.5° is represented on the image plane as a circle of circumference 172 grid units, the radius R of that circle being $172/2\pi = 27$. The west-east extent of 74 degrees of longitude is a chord of that circle of length $2R \sin(74^\circ/2) = 32.5$. Therefore, the number of gridpoints to cover this distance is about 33.

The 23 X 33 gridpoint region of 759 points agrees rather closely with that estimated in Appendix K (op. cit.) of 20 X 23 X 1.6 = 736 points arrived at by measurement on a map.

B. Regional Area

Fig. 2 of Appendix K (op. cit.) indicates that the area over which gridpoint and graphic data would be transmitted to support the Regional Areas for WFO's in the contiguous 48 states and Puerto Rico extends from about 11° to 57° N, the west-east extent being from 140° W to 58° W. The south-north extent of the image plane on a Mercator projection with scale defined at 35° N is

$$IP = \int_{11^\circ}^{57^\circ} \frac{0.819}{\cos\phi}\, r\, d\phi = 0.819r \left[\ln\left(\tan\phi + \frac{1}{\cos\phi}\right) \right]_{11^\circ}^{57^\circ}$$

$$= 5340 \text{ km.}$$

The 80-km gridpoint spacing specified in Appendix K (op. cit.) at 35° N requires 5340/80 = 67 grid units or 68 gridpoints to cover the 11- to 57-degree extent.

At 35° N, the earth distance between 58° W and 140° W is $2\pi r \cos 35^\circ (82/360) = 7468$ km. This requires 7468/80 = 93 grid units or 94 gridpoints to cover the 82 degree longitudinal belt.

This 68 X 94 = 6392 area agrees closely with the 68 X 95 grid estimated in Appendix K (op. cit.) arrived at by measurement on a map. This area also supports the Regional Area for Puerto Rico.

If the Lambert conformal projection were to be used for the 48 states, it is not advantageous to support the Regional Area for Puerto Rico with the same grid. If that were done, there would be a tilt for Puerto Rico about equal to that of the easternmost WFO in the 48 states. Since there is no tie between Regional and Local Areas between the WFO in Puerto Rico and those in the 48 states, there is no reason for them to utilize the same grid. If a separate product is sent for Puerto Rico, the lower boundary for the 48-states' product can be around 16° N at 96° W rather than 11° N. The northernmost extent might need to be slightly north of 57° (at 96° W) to compensate for latitude curvature on the map above Seattle.

In summary, use of a Lambert for supporting the Regional Areas would require some redefinition of grids shown in Fig. 3 of Appendix K (op. cit.). Most likely, a separate grid would be needed for Puerto Rico; the reduction in product size for the large grid would roughly compensate for this and a separate grid would allow a "no tilt" grid for Puerto Rico.

Stimulating America's Progress
1913-1988