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DEVELOPMENT OF AN EXPERIMENTAL SYSTEM TO FORECAST PRECIPITATION USING EMPIRICAL ORTHOGONAL FUNCTIONS

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1. INTRODUCTION

Automated forecasts of probability of precipitation (PoP) for approximately 230 conterminous United States stations based on the Model Output Statistics (MOS) technique (Glahn and Lowry, 1972), have been produced by the National Weather Service since the early 1970's. The precipitation event is defined as the occurrence of $\geq$.01 inches of precipitation in a 6- or 12-h period at a station. In using the MOS approach, there are several statistical methods that can be used to relate the predictand event to output from a numerical model such as the Limited-area Fine Mesh (LFM) model (Gerrity, 1977; Newell and Deaven, 1981). The present operational PoP equations were developed with the Regression Estimation of Event Probability (REEP) statistical model (Miller, 1964) with output from the LFM, as were previous PoP forecast equations. We expect that the LFM will soon be replaced by another model and there may be frequent modifications to that and succeeding models. Under these conditions it is particularly desireable to have a statistical model capable of developing stable regression equations from a limited developmental sample. It has been suggested by Glahn (1962; 1965) that some form of orthogonal functions can be used without overfitting the data. The purpose of this report is to describe an experiment to develop PoP forecast equations through an application of the Empirical Orthogonal Function (EOF) technique (Lorenz, 1956). The experiment had two purposes, first, to compare the accuracy of forecasts from REEP and EOF equations, and second, to determine the effect of the amount of developmental data on the forecast accuracy of REEP and EOF equations.

2. OUTLINE OF THE EXPERIMENT

In the first part of the experiment, we determined the optimum number of predictors to include in REEP and EOF 3-mo winter season (December-February) equation sets for the 12-24 hour period after 0000 GMT. This was done by deriving equation sets where the number of predictor terms varied from one set to another and then choosing the set whose forecasts on independent data produced the lowest Brier score (Brier, 1950). For the EOF equation sets, two parameters are involved: the number of predictors comprising the EOF's and the number of EOF's used to derive the final equations. Once we determined the best REEP and EOF equation sets, we performed paired t-tests (Panofsky and Brier, 1968) on independent data to determine if the differences in Brier scores were significant. We performed this part of the experiment on equations developed for a 3-mo winter season because we also wanted to check the feasibility of using equations developed for 3-mo seasons. The results of a previous experiment (Table 1), showed that forecasts from REEP, 3-mo winter season equations were not as good as the forecasts from REEP, 6-mo cool season (October-March) equations. Our hope was that the EOF approach would improve the accuracy of the 3-mo season equations significantly, but have relatively little impact on the 6-mo season equations.

Due to the results of the first part of the experiment, we then derived 10 sets of 6-mo, cool season REEP and EOF equations for the 12-24 hour period after 0000 GMT. For these equation sets we varied the developmental sample from one to five seasons of data. Forecasts from all 10 of these equation sets were verified on independent data and paired t-tests performed on the Brier score results.

3. COMPARISON OF REEP AND EOF EQUATIONS

a. Predictor and Predictand Data Sets

The developmental sample included nearly six winter seasons (1976-77 through 1981-82) of LFM model output and surface observations from the Techniques Development Laboratory's (TDL's) archive of hourly surface reports for approximately 500 conterminous United States stations. Table 2 shows the potential predictor variables used to develop the REEP and EOF equations. These included model output variables valid for 6-, 12-, 18-, 24-, and 30-h projections. The model output variables for projections $\leq$ 12 hours were unsmoothed and five-point space-smoothed for projections $\geq$ 18 hours. We did not screen any observed weather elements as predictors.

b. Regions

Grouping stations into regions increases the sample size used to develop equations. In the MOS system, stations may be grouped into regions if they exhibit similar characteristics of the predictand in response to output from the numerical model. In particular, for each station, we determined the observed relative frequency of precipitation during the 12-24 h period after both 0000 and 1200 GMT for all cases when the LFM forecast of 12- to 24-h precipitation amount was $\geq$.01 inch and for all cases when the LFM forecast of mean relative humidity was $\geq$65%. We chose precipitation amount and mean relative humidity because, from experience, we've found that these fields are generally the most important predictors for PoP forecasting.

The 66 regions we determined are shown in Fig. 1. In some cases, examination of these regions indicates that the boundaries are not necessarily consistent with geographic boundaries. This is not unusual since our purpose is to group together stations which exhibit similar statistical characteristics based on output from the LFM rather than solely from geographical patterns.

c. Equation Development

In the REEP screening procedure, a subset of effective predictors for use in linear-regression equations is objectively selected from a larger set of potential predictors. The equations developed give estimates of the probabilities of occurrence for a given set of predictands. In PoP, precipitation is divided into two binary predictands: precipitation amount $<$.01 inch and $\geq$.01 inch. The predictands are called binary because in the developmental phase, each predictand was assigned a value of either 1 or 0 in a given case depending on whether or not $\geq$.01 inch of precipitation occurred. The potential predictors were either in binary or continuous form. The use of binary predictors helps to account for non-linear relationships. A description of the screening procedure can be found in Glahn and Lowry (1972).

The EOF approach also makes use of the REEP screening procedure to objectively select a subset of predictors from a larger set of potential predictors, and EOF's are determined from this subset of effective predictors. The EOF's are then used as a new set of potential predictors by the REEP screening procedure which objectively selects a subset of the EOF's for use in linear regression equations. Our approach is different from the one described by Lorenz (1956) in three ways. First, to determine the EOF's, we use as predictors forecast values of many different variables at a single location, whereas, forecast values of one variable at many points over a large geographical area were used as predictors by Lorenz. Second, through the use of a screening procedure we select from a larger set of potential predictors a smaller set of predictors to use as variables to determine EOF's; Lorenz used the entire potential predictor set. Finally, the EOF's that are determined are used as potential predictors and also submitted to the screening procedure to objectively select a subset of the EOF's for use in linear regression equations. Lorenz did not screen the EOF's, instead, the first few were taken in order.

For the purpose of determining the best REEP and EOF predictors, we developed separate sets of equations where we varied the number of predictor terms. For each set, we combined data from all stations within a region and developed equations for the 12-24 h period after 0000 GMT. REEP equation sets were developed with 4, 6, 8, 10, 12, 14, 16, 18, and 20 predictor terms. For EOF equations, as previously discussed, we varied the number of EOF's used to derive the linear-regression equations as well as the number of predictors defining each EOF to determine the best EOF equation. We developed EOF's using 10, 15, and 20 predictors as variables. Separate sets of equations were developed using 1, 2, 3, 4, 6, and 8 of the 10-variable EOF's. From the EOF's based on 15 variables, we developed separate sets of equations using 1, 2, 3, 4, 6, 8, and 10 EOF's. Similarly, from the EOF's based on 20 variables we used 1, 2, 4, 6, 8, 10, and 12 EOF's to derive equations. The most important predictors in both the REEP and EOF equations were LFM forecasts of mean relative humidity, precipitation amount, dew-point depression, and moisture convergence.

d. Verification Results

For the REEP and EOF equation sets, we performed a comparative verification on independent data combined from 218 stations for the period December 1982 through February 1983. As part of the verification, we calculated the Brier score for each equation set. The scores for the different REEP and EOF equation sets are shown in Tables 3 and 4, respectively. Table 3 indicates that the best REEP equation set in terms of Brier score was the 18-term equation set. However, there was little difference in scores among the equation sets with 10 terms or more. Table 4 indicates that the best EOF equation set in terms of the Brier score was the one using eight EOF's where the EOF's were based on 20 variables each. Closer examination of Tables 3 and 4 reveals that the EOF equations based on 10 variables did not produce better results than the EOF 18-term REEP equation. In fact, the best EOF equation set based on 10 variables was only better than REEP equations using eight terms or less. Also, the EOF's based on 15 and 20 variables did better than the 18-term REEP equation set with as few as three or four EOF's.

A paired t-test comparison between the Brier scores for the best REEP and the best EOF equation sets indicated that the difference between REEP and EOF scores were not significant, even at the 10.0% level. In the previous experiment comparing 3- and 6-mo season REEP equations discussed in Section 2, the equations included 12 terms each, so we also compared the 12-term REEP equation set with the best EOF equation set. Although the difference in Brier scores between the two was significant to the 5.0% level, the difference between 3- and 6-mo season 12-term equations as shown in Table 1 was even greater. Hence, as a result of this part of the experiment, we decided to no longer experiment with the 3-mo winter season, and that in the next part of the experiment, the EOF equations would be derived with eight EOF's based on 20 variables each.

4. COMPARISON OF REEP AND EOF EQUATIONS WITH DIFFERING AMOUNTS OF DEVELOPMENTAL DATA

a. Equation Development

For the second part of the experiment we derived REEP and EOF equation sets for the 6-mo cool season (October-March). Our developmental sample consisted of five cool seasons (1977-78 through 1981-82) of LFM model output and surface observations from TDL's archive of hourly surface reports for the same stations as before. We used the same potential predictor variables as before (Table 2) to develop the equations. The regions we used are the 26 regions shown in Fig. 2. These are the regions currently used by the operational PoP forecast system (National Weather Service, 1980) and were determined in the same manner as previously discussed.

For the purpose of determining the effect of the amount of data on the accuracy of REEP and EOF equations, we developed separate sets of equations in which we varied the length of the developmental sample. Specifically, for the first pair of REEP and EOF equation sets, we used only the 1981-82 cool season data to derive equations. For the second pair, we used the 1980-81 and the 1981-82 cool seasons. For the third pair, we used the 1979-80 through 1981-82 cool seasons. For the fourth pair, we used the 1978-79 through 1981-82 cool seasons. Finally, for the fifth pair, we used all five cool seasons of data. For each set, we combined data from all stations within a region and developed equations for the 12-24 h forecast period after 0000 GMT.

The REEP equations for this part of the experiment were developed with 12 terms in order to keep them consistent with the operational PoP forecast equations. Also, we used 12 terms because we consider 18 terms too many to use for stable PoP equations with small samples of data. The EOF equations were derived from eight EOF's that were based on 20 variables each. Past experience has shown that 12 terms, even for the small data samples, is a good estimate for the optimum number of predictors to use for developing REEP equations. However, the same assumption can't be made for the EOF equations. We found eight EOF's based on 20 variables each to be the optimum number of predictors for equation development in the first part of the experiment, but this combination may or may not be optimum for all the EOF equations developed for this part of the experiment, especially for small data samples.

As with the 3-mo winter season equations, the LFM forecasts of mean relative humidity, precipitation amount, dew-point depression, and moisture convergence were the most important predictors. Table 5 shows a comparison of the reduction of variance between REEP and EOF equations that were developed with two cool seasons of data for the 12-24 h period from 0000 GMT for region 20 of Fig. 2. Table 5 reveals that the first EOF chosen produced a reduction of variance that was much larger than that produced by the first predictor chosen for the REEP equation. Also, the first EOF accounted for more than 95% of the total reduction of variance produced by the entire REEP equation, and the total reduction of variance by the EOF equation was greater than that associated with the REEP equation.

b. Verification Results

For the REEP and EOF equation sets, we performed comparative verifications on independent data combined from 218 stations for the period October 1982 through March 1983. The Brier scores for the different REEP and EOF equation sets are shown in Table 6. Table 6 indicates that, as one might expect, the scores for REEP and EOF equations improved as each additional season of data was added. A direct comparison of scores between REEP and EOF equations that were developed on the same data reveals that, with one season of data, there was little difference between the scores for REEP and EOF; although EOF scores are slightly better. As more data were added, the improvement of EOF over REEP increased.

As before, we performed paired t-tests to determine whether the differences between Brier scores for the REEP and EOF equation sets were significant. We tested for significance to the 10.0%, 5.0%, 1.0%, and 0.1% levels. The results, also shown in Table 6, indicate that the improvements due to the addition of a second and a third year of data were to the highest level of significance (the 0.1% level) for both REEP and EOF. The addition of data beyond three seasons also improved the results, but less significantly (to the 5.0% level or less). The results of t-tests comparing REEP and EOF equations developed on equal amounts of data showed that the differences between REEP and EOF scores were not significant at any level with two seasons of data or less, but the EOF scores showed significant improvement over REEP, at the 5.0% level, with three and five seasons of data, and at the 1.0% level with four seasons of data.

5. SUMMARY

We conducted an experiment to determine the usefulness of using EOF's to derive MOS PoP forecast equations and to compare them to equations developed on exactly the same data by the REEP technique. We found that equations developed through an application of EOF's can provide significantly more accurate forecasts than REEP equations when both are developed on exactly the same data sample, if enough data are available. In this experiment, we varied the amount of data used to develop the equations by varying the size (number of seasons) of the developmental sample. This produced significant improvement of EOF's over REEP when three or more 6-mo seasons (18, 24, or 30 months) of data were used, but only slight improvement when two seasons or less (6 or 12 months) of data were used. Because the PoP forecast system is a regionalized one, we will experiment further where we will decrease the number of regions as a means of increasing the data sample for each equation when two seasons, or less, of data are used. Also, in this experiment we did not determine the optimum combination of EOF's and variables for EOF equation development for the smaller data sets, but for the next experiment we plan on doing so for data sets with 12 months of data or less. If the results from this future experiment indicate that EOF equations can also be significantly better than REEP equations with only one or two 6-mo seasons of data, then we will adopt the EOF approach as the statistical technique to develop future MOS PoP forecast systems.

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Table 1. Brier scores for the 3- and 6-mo season REEP equation sets for PoP forecasts for the 12-24 and 36-48 h periods from 0000 and 1200 GMT. The sample consisted of independent data combined from 218 stations for the period December 1982 through February 1983. The percent improvement of 3-mo equations over 6-mo equations is also shown. The sample included approximately 17,000 cases for each period. The scores shown below are for equations which did include observed predictors as variables. Note that this is not the case for any of the other equations sets developed for this experiment.

System	Period			
	0000 GMT		1200 GMT	
	12-24 h	36-48 h	12-24 h	36-48 h
3-mo	.1835	.2319	.1807	.2269
6-mo	.1814	.2317	.1802	.2238
% Improvement 3-mo/6-mo	-1.2	-0.1	-0.3	-1.4

Table 2. The potential predictors included in the development of all the experimental PoP equations.

Definition	Levels
a. Model Output Predictors	
East-west wind component	200 mb, 500 mb, 700 mb, 850 mb, 1000 mb
North-south wind component	200 mb, 500 mb, 700 mb, 850 mb, 1000 mb
Mean relative humidity	SFC-500 mb
Vertical Velocity	500 mb, 700 mb, 850 mb
Precipitable Water	SFC-500 mb
Precipitation Amount	--
b. Model Output Derived Predictors	
Temperature-dew point depression	500 mb, 700 mb, 850 mb
Temperature Advection	500 mb, 700 mb, 850 mb
Vorticity Advection	500 mb, 700 mb, 850 mb, 1000 mb
Thickness	1000-500 mb, 1000-850 mb, 850-500 mb
Relative Vorticity	500 mb, 700 mb, 850 mb, 1000 mb
Moisture Convergence	500 mb, 700 mb, 850 mb
K Index	--
G Index	--
Total-Totals Index	--
c. Geoclimatic Predictors	
Sine of the day of the year	--
Cosine of the day of the year	--
Station latitude	--
Station longitude	--
Station elevation	--

Table 3. Brier scores for the 3-mo season REEP equation sets for PoP forecasts for the 12-24 h period after 0000 GMT. The sample consisted of independent data combined from 218 stations for the period December 1982 through February 1983 and included 19010 cases for each set.

No. of Terms	Brier Score
4	.2050
6	.2008
8	.1984
10	.1975
12	.1976
14	.1974
16	.1971
18	.1970
20	.1971

Table 4. Same as Table 3 except for EOF equation sets based on 10, 15, and 20 variables.

No. of EOF's	Brier Score		
	10 Variables	15 Variables	20 Variables
1	.2037	.2029	.2024
2	.2008	.1988	.1977
3	.1989	.1963	--
4	.1987	.1967	.1964
6	.1985	.1974	.1971
8	.1977	.1968	.1960
10	--	.1969	.1968
12	--	--	.1968

Table 5. Reduction of variance produced by each predictor term and EOF for the REEP and EOF equation sets, respectively, developed with two cool seasons of data for region 20 (see Fig. 2) for the 12-24 h period after 0000 GMT. "Cont." under Binary Threshold means the predictor is continuous, not binary.

REEP					EOF	
Predictor Term	Level	Projection (h)	Binary Threshold	Reduction of Variance	EOF Number	Reduction of Variance
Mean Rel. Hum.	--	18	Cont.	.3635	1	.4842
Precip. Amt.	--	12-24	Cont.	.0513	4	.0002*
Precip. Amt.	--	12-24	Cont.	.0195	2	.0003*
Precip. Amt.	--	12-24	.00075 m	.0298	3	.0129
Dew-point Dep.	700-mb	18	2°C	.0098	8	.0000*
Rel. Vort.	500-mb	18	Cont.	.0146	10	.0075
Mean Rel. Hum.	--	12	90%	.0025	11	.0023
Moisture Conv.	700-mb	12	Cont.	.0035	20	.0020
Moisture Conv.	850-mb	24	Cont.	.0009		
Mean Rel. Hum.	--	24	70%	.0010		
Precip. Amt.	--	12-24	.00075 m	.0017		
Mean Rel. Hum.	--	12	70%	.0006		
			Total	.4987	Total	.5094

*We developed the 12-24 h PoP equations simultaneously with 12-18 h and 18-24 h equations in order to be consistent with past PoP forecast equation developments. As a result, the reductions of variance for some terms are small because they were chosen as important predictors for one of the 6-h periods and were not important for the 12-h period.

Table 6. Verification results for PoP forecasts for the 12-24 h period from 0000 GMT for REEP and EOF equation sets using various amounts of data. The sample consisted of independent data combined from 218 stations for the period October 1982 through March 1983. The digit appended to "REEP" and "EOF" indicates the number of seasons of data equations were developed on. A t-test score of >3.29 indicates a significant improvement in terms of Brier score of the equation set listed across the top of the table over the equation set listed down the left-most column of the table to the 0.1% level, >2.58 to the 1.0% level, >1.96 to the 5.0% level, and >1.65 to the 10.0% level. We have highlighted the >1.0% and 5.0% levels by double and single asterisks, respectively. The scores and t-tests were based on a matched sample of 36567 cases. The long dash (--) indicates a paired t-test was not conducted for this particular combination of equation sets.

		Paired t-test scores									
Equation Type	Brier Score	REEP1 vs.	REEP2 vs.	REEP3 vs.	REEP4 vs.	REEP5 vs.	EOF1 vs.	EOF2 vs.	EOF3 vs.	EOF4 vs.	EOF5 vs.
REEP1	.1999	--	7.51**	--	--	--	0.57	--	--	--	--
REEP2	.1932	--	--	3.60**	--	--	--	1.28	--	--	--
REEP3	.1912	--	--	--	1.42	--	--	--	2.38*	--	--
REEP4	.1903	--	--	--	--	2.56*	--	--	--	3.24**	--
REEP5	.1893	--	--	--	--	--	--	--	--	--	2.52*
EOF1	.1995	--	--	--	--	--	--	6.59**	--	--	--
EOF2	.1925	--	--	--	--	--	--	--	3.83**	--	--
EOF3	.1901	--	--	--	--	--	--	--	--	2.61*	--
EOF4	.1886	--	--	--	--	--	--	--	--	--	1.18
EOF5	.1880	--	--	--	--	--	--	--	--	--	--

Figure 1. Regions used to develop REEP and EOF equations for the 3-mo winter season.

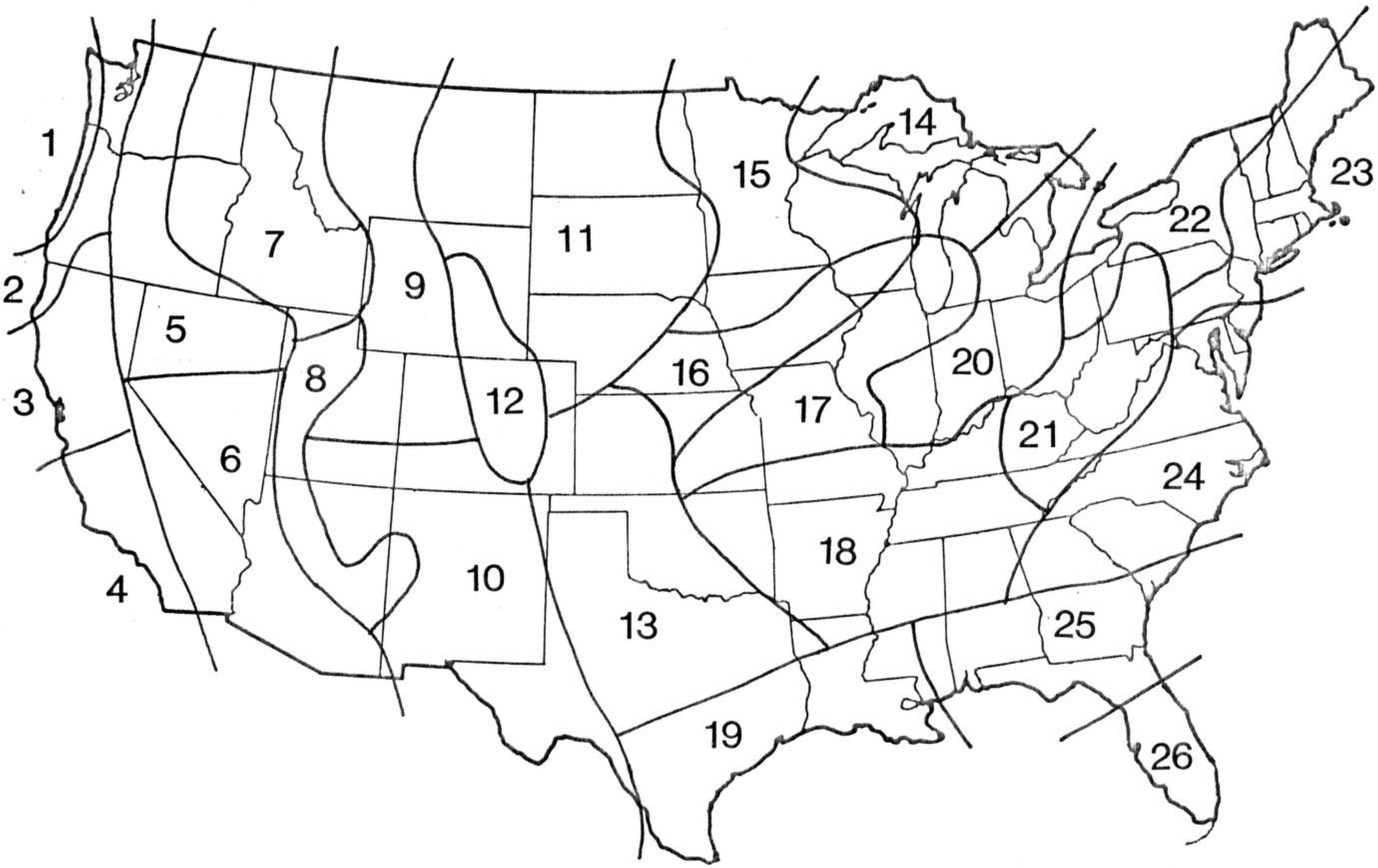

Figure 2. Same as Fig. 1 except for the 6-mo cool season.