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AN IMPROVED USE OF THE LOGIT MODEL TO TRANSFORM PREDICTORS FOR PRECIPITATION TYPE FORECASTING

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1. INTRODUCTION

A new system for forecasting the conditional probability of precipitation type (PoPT) (Bocchieri, 1978) became operational in September 1978 within the National Weather Service. In the PoPT system, we used the Model Output Statistics (MOS) technique (Glahn and Lowry, 1972) with output from the Limited-area Fine Mesh (LFM) model (National Weather Service, 1971; Gerrity, 1977) to develop probability forecast equations for three precipitation type categories: frozen (snow or ice pellets), freezing (freezing rain or drizzle), and liquid (rain or mixed types). The PoPT system evolved from an earlier operational system for forecasting the probability of frozen precipitation (PoF) (Glahn and Bocchieri, 1975; Bocchieri and Glahn, 1976).

In the PoPT and PoF systems, we needed to combine data from different stations to develop the forecast equations because of the limited amount of developmental data. To justify combining the data, we transformed the predictor variables into deviations from 50% values. The 50% value of a variable is that value which indicates a 50-50 chance of frozen precipitation at a station, provided precipitation occurs. In the PoPT system, the forecasts for the freezing precipitation category are also a function of predictors transformed in this manner, even though the 50% values give the probability of frozen precipitation as opposed to all other precipitation types (see Bocchieri, op. cit., for more details). We determined the 50% values for each model output predictor and for each station by using the logit model (Brelsford and Jones, 1967; Jones, 1968) to fit the data. The logit model provides a means of fitting a sigmoid or S-shaped curve when the dependent variable is binary and the independent variable is continuous.

As discussed in Glahn and Bocchieri, op. cit., the 50% value of a variable can vary quite a bit from station to station depending on local factors, especially station elevation. Our assumption was that a given deviation of a predictor from its 50% value would produce the same probability of frozen precipitation at different stations. This assumption would be exact if the logit curve for a given predictor had the same shape for each station. Actually, this isn't true; that is, for a given predictor, some curves are quite steep while others are quite shallow. For example, for a steep logit curve, the difference in the 850-mb temperature (850 T) between the 5% and the 95% points of the curve might be 4 K; however, for a shallow logit curve, this difference might be 8 K.

We hypothesized that we could improve the accuracy of the probability forecasts in the PoPT system by transforming predictors to account not only for the difference in 50% values between stations but also for the difference in steepness or spread of the logit curves. In this paper, we'll describe our efforts to develop an improved transformation procedure with respect to forecasting frozen precipitation.

Any improvement realized should be beneficial not only for the probability of frozen precipitation forecasts in the PoPT system but also for the probability of freezing precipitation. First, in section 2, some mathematical properties of the logit model are presented. In section 3, we describe and compare the new transformation procedures tried.

2. SOME PROPERTIES OF THE LOGIT MODEL

The logit model provides a means of fitting an S-shaped, symmetric curve when the dependent variable (Y) is binary and the independent variable (X) is continuous. The probability of the binary variable having the value of one, say, can be expressed

$$P\ (Y = 1|X) = \frac{\exp\ (\alpha + \beta X)}{1 + \exp\ (\alpha + \beta X)} \tag{1}$$

In our application, Y takes the value one (zero) for an observation of frozen (unfrozen) precipitation; also, the probability is conditional on the event that precipitation occurs. Throughout the rest of this paper, we'll use the word "snow" instead of "frozen precipitation" and the expression P(Snow) instead of $P(Y = 1|X)$.

The computer program we use (Jones, op. cit.) determines maximum likelihood estimates for the model parameters α and β. As discussed in Cox (1970), the logit model has the following properties:

1. The value of X at which P(Snow) is 50% is given by $-\alpha/\beta$.

2. The β parameter measures the steepness or slope of the logit curve; the larger the absolute value of β, the steeper the curve.

3. The β parameter is such that $1/\beta$ is approximately the distance in X units between the 75% point and the 50% point of the curve. Also, the distance between the 95% point and the 50% point is approximately $3/\beta$. Below, we derive the fact that this later distance is actually equal to $3.047/\beta$.

We can write (1) as

$$P(\text{Snow}) = \frac{1}{1 + \exp - (\alpha + \beta X)}\ , \tag{2}$$

from which we obtain

$$\alpha + \beta X = \ln \left[\frac{P(\text{Snow})}{1 - P(\text{Snow})} \right], \tag{3}$$

or

$$\alpha + \beta X = \ln \left[P(\text{Snow}) \right] - \ln \left[1 - P(\text{Snow}) \right]. \tag{4}$$

Now, let $X_{.95}$ ($X_{.50}$) be the value of X at which P(Snow) is 95% (50%). Then, from (4) we obtain

$$\alpha + \beta X_{.95} = 3.047, \tag{5}$$

and

$$\alpha + \beta X_{.50} = 0.0. \tag{6}$$

Subtracting (6) from (5), we get

$$X_{.95} - X_{.50} = 3.047/\beta. \tag{7}$$

In this paper, we define the location (LOC), SLOPE, and SPREAD parameters of a logit curve as

$$\text{LOC} = X_{.50}, \tag{8}$$

$$\text{SLOPE} = \left| \beta \right|, \tag{9}$$

and

$$\text{SPREAD} = \left| 3.047/\beta \right|. \tag{10}$$

The LOC parameter is so called because it locates the mid-point (50% point) of the logit curve with respect to the axis which defines the X variable. The SLOPE and SPREAD parameters characterize the shape of the curve.

The LOC, SLOPE, and SPREAD parameters are illustrated in Fig. 1 which shows logit curves for Fort Smith, Arkansas and Sheridan, Wyoming. For these stations, P(Snow) is shown as a function of the LFM 850 T forecast. The developmental data sample for these curves consisted of 5 winter (September through April) seasons, 1972-73 through 1976-77. For each station, we matched 850 T forecasts from the LFM model and corresponding surface observations of precipitation type for seven projections--6, 9, 12, 15, 18, 21, and 24 hours. The data from all projections and from both the 0000 GMT and 1200 GMT LFM cycle times were combined into one sample so that as many snow cases as possible would be included. We then fit the logit model to the data and computed the LOC, SLOPE, and SPREAD for each station from (8), (9), and (10), respectively. In Fig. 1, note that not only does the LOC of the curves differ but so do the SLOPE and SPREAD. The logit curve for Fort Smith is steeper (SLOPE = 1.19) than the curve for Sheridan (SLOPE = 0.34), and therefore, the SPREAD of the curve for Fort Smith, 2.56, is less than that for Sheridan, 8.91. Obviously, if the 850 T is transformed into deviations from 50% values, then a given deviation would not give the same probability of snow at Fort Smith and Sheridan. In fact, a −2 K deviation gives about a 90% chance of snow at Fort Smith but only about a 65% chance at Sheridan. This illustrates the need to account not only for the difference in the LOC between the curves but also for the difference in the SLOPE or SPREAD.

The variation of the LOC parameter between stations has been well documented by Glahn and Bocchieri, op. cit., and Bocchieri and Glahn, op. cit. In Fig. 2, an analysis of the absolute value of the SLOPE parameter for the 850 T logit curves is shown for the conterminous United States; the logit curve for each station was derived from the same sample used to obtain the curves in Fig. 1. Because of a lack of snow cases, we couldn't objectively determine the logit curves for the MOS stations below the dashed line; therefore, we didn't extend the analysis to those stations. The analysis shows that the SLOPE's for stations in the Rocky Mountain region are generally less than the SLOPE's for stations to the east and west of that region. In the Rocky Mountain region, the SLOPE's are generally < .5; in the eastern half of the United States and near the Pacific-Northwest coast, the SLOPE's are generally between .6 and .9.

3. PREDICTOR TRANSFORMATION METHODS

Because of the differences in the LOC, SLOPE, and SPREAD parameters of the logit curves from station to station, we need to transform the predictors in some manner in order to combine data from different stations. In this section, we compare three transformation procedures called the centered, standardized, and linearized procedures.

In the centered procedure,

$$X_T = X - LOC, \qquad (11)$$

where X_T is the transformed variable, and X is the original variable. In this manner, the difference in the LOC parameter of the logit curves from station to station is accounted for. The centered procedure was used in the development of the operational PoPT and PoF systems. In the standardized procedure,

$$X_T = \frac{X - LOC}{SPREAD} \; . \qquad (12)$$

That is, the original variable is transformed such that the differences in the LOC and SPREAD parameters of the logit curves from station to station are accounted for. In the linearized procedure,

$$X_T = \frac{\exp\ (\alpha + \beta X)}{1 + \exp\ (\alpha + \beta X)} \; . \qquad (13)$$

That is, the transformed variable is obtained by applying the logit function to the original variable. All the information in the logit curve is therefore included in X_T, and the relationship between X_T and the actual relative frequency of snow should be linear, if the logit model is appropriate.

Now, we'll illustrate and compare the three transformation procedures on the developmental data sample for the case of one predictor, the 850 T. We developed three generalized-operator systems to forecast the conditional probability of snow for the 18-h projection from the 0000 GMT LFM cycle time. In each system, the 850 T was included in transformed form. The three systems are called the centered, standardized, and linearized systems to correspond to the transformation procedure used in each. To develop each system, we combined data from 174 MOS stations; the developmental data period was the same as that used for Figs. 1 and 2.

For example, in the centered system, (11) was used to transform the 850 T predictor at each station for each precipitation case in the sample; similarly, (12) and (13) were used, respectively, for the standardized and linearized forecast systems. After transforming the predictors and combining data from all stations, we used the logit model to develop the forecast equations for the centered and standardized systems. For the linearized system, we used the Regression Estimation of Event Probability (REEP) (Miller, 1964) technique. REEP is essentially linear regression with a binary predictand; linear regression seemed to be appropriate for the linearized system since the relationship between the transformed variable and the predictand should be linear. The generalized-operator forecast equations for the centered, standardized, and linearized forecast systems are, respectively,

$$P(\text{Snow}) = \frac{1}{1 + \exp\ (-.397 + .612\ X)}\ , \tag{14}$$

$$P(\text{Snow}) = \frac{1}{1 + \exp\ (-.441 + 3.01\ X)}\ , \tag{15}$$

and

$$P(\text{Snow}) = .026 + .999\ X, \tag{16}$$

where X is the 850 T appropriately transformed.

The Brier score (Brier, 1950) and the reliability were computed for each forecast system for each of three groups of stations and for all stations combined. We determined the groups by stratifying the stations according to the SLOPE parameter of the 850 T logit curve (see Fig. 2). Groups A, B, and C consisted, respectively, of stations whose logit curves had $|\text{SLOPE}| < .4$, $.5 \leq |\text{SLOPE}| \leq .7$, and $|\text{SLOPE}| > .8$[1]. The groups were purposely chosen to illustrate the effect of not accounting for the differences in the SLOPE's of logit curves. The Brier scores, shown in Table 1, indicate that the standardized system was the best of the three for each group and for all stations combined. Note that the standardized system showed more improvement over the centered system in groups A and C than in group B. The reason is that the $|\text{SLOPE}|$ for group B stations (between .5 and .7) is similar to that for the generalized-operator

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[1] Note that stations with $.4 \leq |\text{SLOPE}| < .5$ and $.7 < |\text{SLOPE}| \leq .8$ were omitted.

logit forecast equation (14), about .6; but the |SLOPE| for stations in groups A and C are quite a bit different than .6. The linearized system was worse than the centered system for stations in group B and for all stations combined.

In addition to the Brier score, we examined the reliability of the probability forecasts for the three forecast systems. Figs. 3 and 4 show the reliability results for all stations combined and for group A, respectively. In Fig. 3, we can see that the probability forecasts for the centered and standardized systems were quite reliable with little difference between them; however, the reliability of the linearized system was not as good. The result of not accounting for the variation of the SLOPE of the logit curves between stations is more evident in Fig. 4; that is, for stations with $|SLOPE| < .4$, the reliability of the centered system was clearly worse than that of the standardized and linearized systems. Although, even for the latter two systems, the reliability was not very good in the range 0% to 50%. Since the SLOPE's of the curves for stations in this group are less than the average SLOPE for all stations combined (about .6), the probability forecasts from the centered system were too low in the range 0% to 50% and generally too high in the range 50% to 100%. The forecasts from the other two systems also exhibited this bias but to a lesser extent.

Based on the above verification on the developmental sample, we decided to make further comparisons between the centered and standardized systems and concluded that the linearized system didn't perform well enough to be considered further for operational implementation.

In order to further compare the centered and standardized forecast systems, we redeveloped the generalized-operator forecast equations with three predictors: 850 T, 1000-500 mb thickness, and boundary layer wet-bulb temperature. These predictors account for most of the useful information from the LFM model with respect to PoF forecasting. The developmental data sample and the predicor transformation procedures were the same as those for the one-predictor systems described above.

We verified the new centered and standardized systems on both developmental and independent data samples; the independent sample consisted of data from September 1977 through February 1978. The verification was done for three groups of stations and for all stations combined; the groups were again determined by the SLOPE of the logit curves for the 850 T. Groups A, B, and C consisted, respectively, of stations with $|SLOPE| < .5$, $.5 \leq |SLOPE| \leq .7$, and $|SLOPE| > .7$. Note that the threshold values for these SLOPE's are slightly different than those used for the groups in Table 1.

The Brier scores for both the developmental and independent data samples and the improvements in Brier score of the standardized system over the centered system are shown in Table 2. The results indicate that the standardized system was better than the centered system for each group, for all stations combined, and for both the developmental and independent data samples. The margin of improvement was about 2% for all stations combined. Note that the improvement was generally greater for groups A and C than for group B; the reason for this is similar to that given for the results in Table 1.

4. SUMMARY AND CONCLUSIONS

A new MOS system for forecasting the conditional probability of precipitation types, called PoPT, became operational within the National Weather Service in September 1978. In the PoPT system, we needed to combine data from different stations to develop the forecast equations because of the limited amount of developmental data available. To justify combining data from different stations, we transformed the predictor variables into deviations from "50% values"; the 50% value of a variable is that value which indicates a 50-50 chance of frozen precipitation at a station, provided that precipitation occurs. We call this transformation procedure the centered procedure. The 50% values were determined for each LFM model output variable and for each station with the logit model. Our assumption was that a given deviation of a variable from its 50% value would produce the same probability of frozen precipitation at any station.

Although all logit curves are S-shaped, we found that the spread or slope of the curves can vary significantly from station to station. Therefore, the assumption underlying the centered transformation procedure is not totally satisfactory; that is, a given deviation of a variable from its 50% value doesn't give the same probability of frozen precipitation at all stations.

We experimented with two other predictor transformation procedures called the standardized and linearized procedures, as candidates to replace the centered procedure. In the standardized procedure, we transformed a predictor by subtracting its 50% value and dividing by a measure of the spread of its logit curve. In the linearized procedure, the transformed predictor was the probability of frozen precipitation as given by the logit function.

We did a comparative verification on the developmental data sample between generalized-operator forecast systems developed with the centered, standardized, and linearized transformation procedures. Each system included one predictor, the 850-mb temperature. The verification results indicate that the standardization system was the best of the three and that the linearized system was the worst. Based on these results, we decided to eliminate the linearized procedure from further consideration and to make more comparisions between the centered and stardardized procedures.

To further compare the centered and standardized procedures, we redeveloped the generalized-operator forecast systems and included three predictors; 850-mb temperature, 1000-500 mb thickness, and boundary layer wetbulb temperature. From verification results for both developmental and independent data samples, we concluded that the standardized procedure was better than the centered procedure by about 2% in the Brier score. We plan to use the new transformation method when we redevelop the PoPT forecast system. Even though we've shown that the standardized transformation procedure should improve the probability forecasts for the frozen category in the PoPT system, a similar improvement should be realized for the freezing precipitation forecasts; the reason is that the latter forecasts are a function of predictors which are transformed with respect to the frozen category.

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Table 1. The Brier scores for the centered, standardized, and linearized forecast system with one predictor, the 850 T. The scores were computed for the developmental data sample for three groups of stations and for all stations combined as defined in the text. The projection is 18 hours from the 0000 GMT LFM cycle time. The number of cases for each group is shown in parentheses.

Forecast System	Brier Scores			
	Group A (1159 Cases)	Group B (13214 Cases)	Group C (2533 Cases)	All Stations (23292 Cases)
Centered	.234	.124	.067	.121
Standardized	.216	.123	.063	.119
Linearized	.218	.127	.065	.122

Table 2. The Brier scores for conditional probability of snow forecasts made from the centered and standardized systems which include three predictors. The station groups are defined in the text. The forecast projection is 18 hours from the 0000 GMT LFM cycle. The number of cases is shown in parentheses. D=developmental data (September-April, 1972-73 through 1976-77); I=independent data (September-February, 1977-78).

Forecast System	Brier Score							
	Group A		Group B		Group C		All Stations	
	D (2409)	I (470)	D (13214)	I (2767)	D (7669)	I (1500)	D (23292)	I (4737)
Centered	.156	.075	.111	.092	.082	.068	.106	.083
Standardized	.153	.073	.110	.090	.080	.067	.104	.081
% Improvement	1.9	2.7	0.9	2.2	2.4	1.5	1.9	2.4

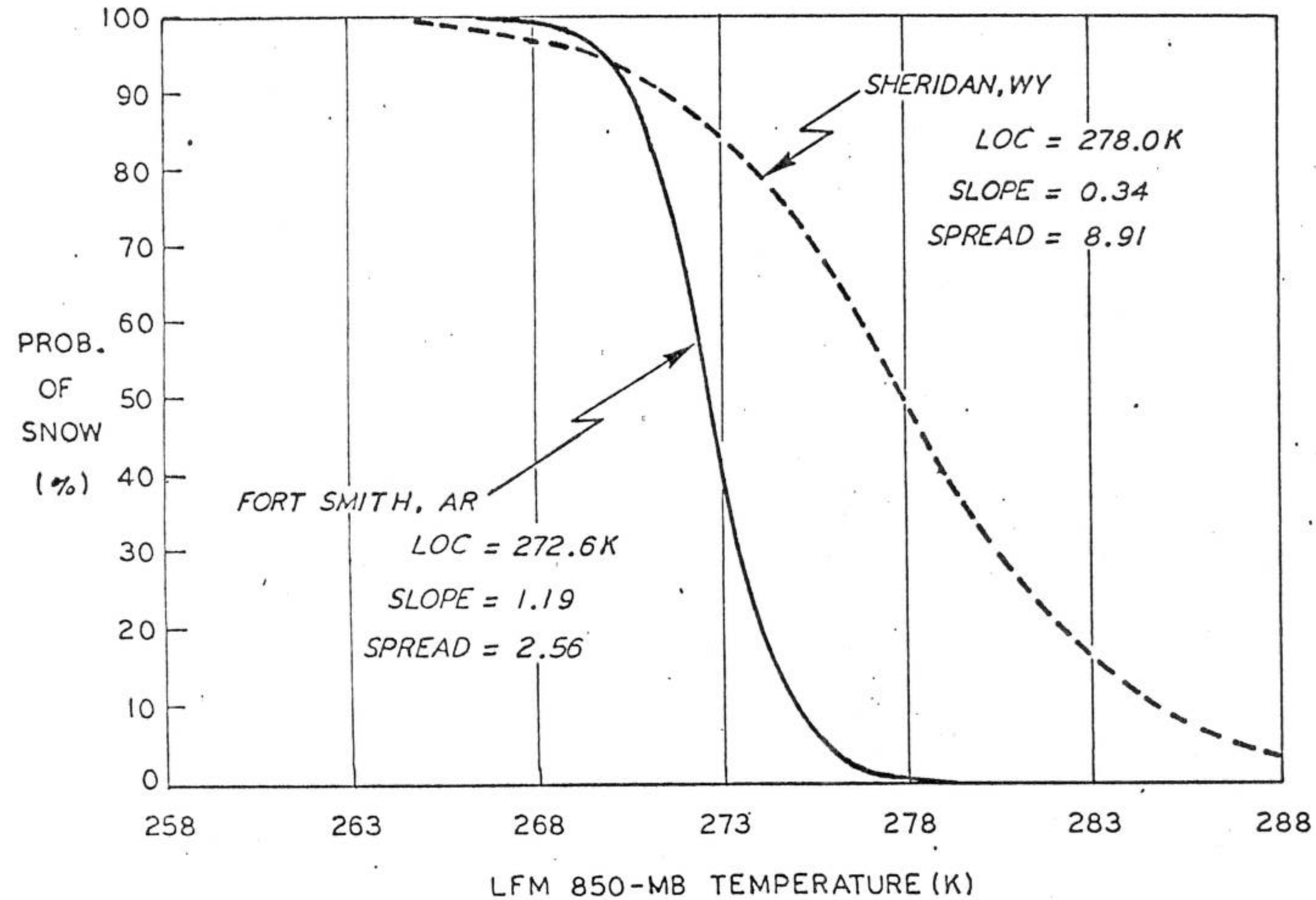

Figure 1. The logit curves for Fort Smith, Arkansas and Sheridan, Wyoming. The curves give the conditional probability of snow as a function of the LFM 850 T forecast and were derived with data from 5 winter seasons, 1972-73 through 1976-77. The LOC, SLOPE, and SPREAD parameters are discussed in the text.

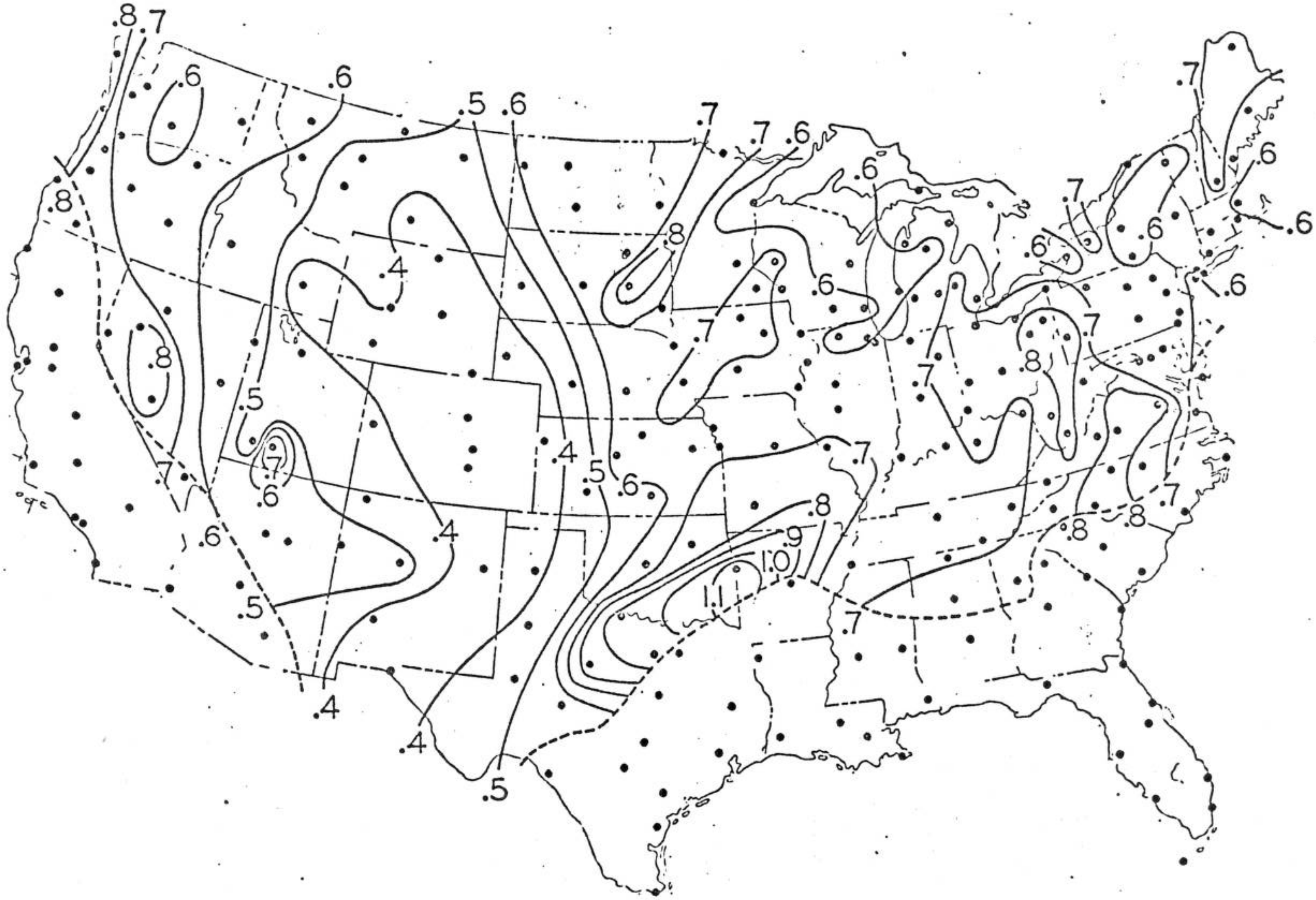

Figure 2. An analysis of the SLOPE of the 850-mb temperature logit curves. The logit curve for each MOS station was developed from the same sample used for the curves in Fig. 1. The logit curves for the stations below the dashed line couldn't be determined objectively; therefore the analysis is not shown there.

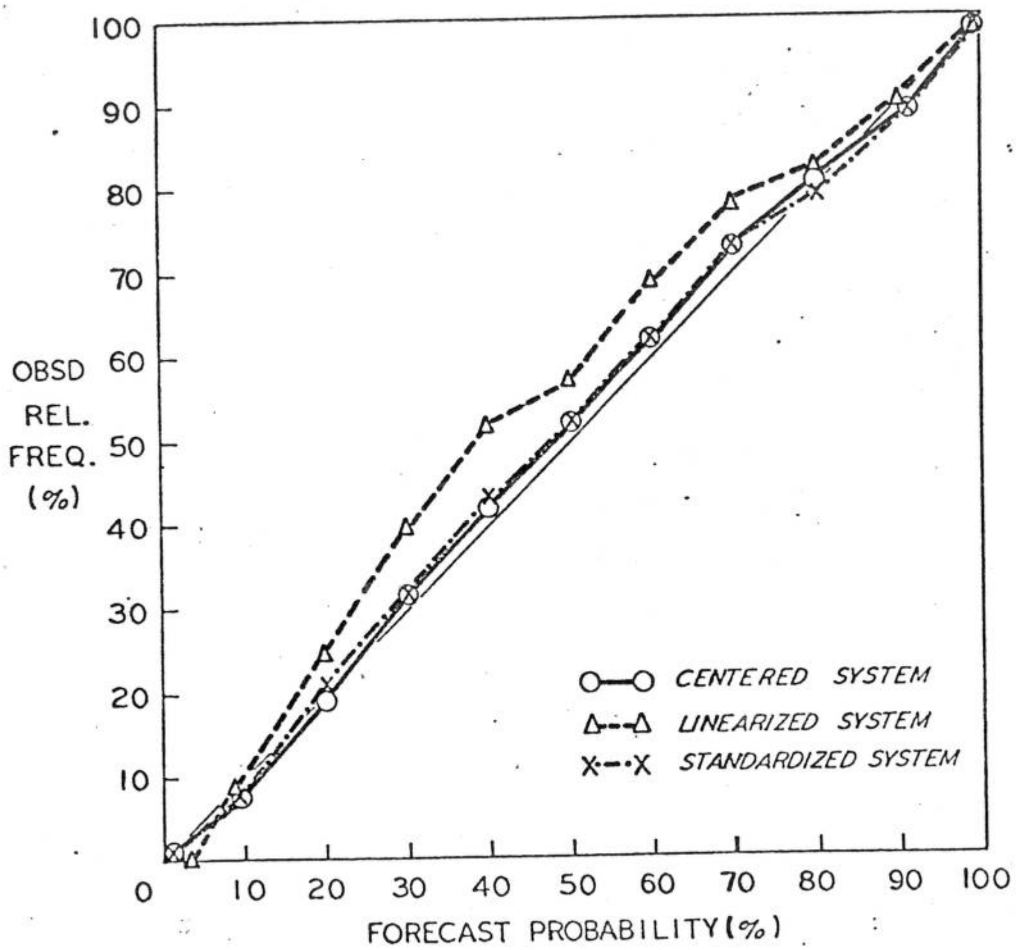

Figure 3. The reliability of the centered, standardized, and linearized forecast systems for all stations combined for the developmental data sample. The forecast equations include one predictor, the 850 T. 18-h projection from the 0000 GMT LFM cycle time.

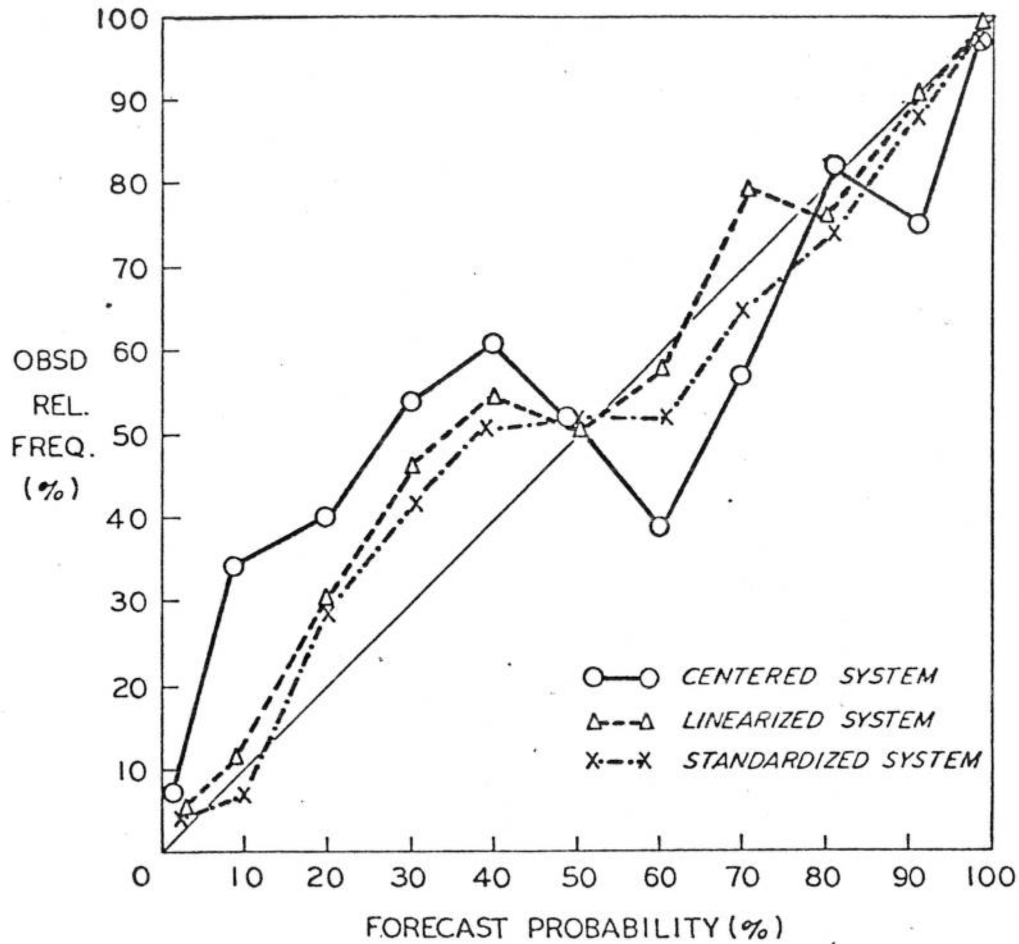

Figure 4. The same as Fig. 3 except that the reliability is shown for stations whose 850 T logit curves have $|\text{SLOPE}| < .4$.