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# A simple model for freezing rain ice loads

Kathleen F. Jones *

*USACRREL, 72 Lyme Road, Hanover, NH 03755-1290, USA*

**Abstract**

There are many models for hindcasting ice loads from meteorological data measured during freezing rain storms. Each model is based on the physics of the ice accretion process and on empirical observations. However, these models predict significantly different ice loads for the same freezing rain storm, making it difficult to use model results to determine design ice loads. In this paper, we describe a simple ice load model that can be used to make conservative back-of-the-envelope calculations of ice loads based on the precipitation rate and wind speed. Using historical weather data from Springfield, IL, we compare the ice loads from this model with those from other models and discuss the reasons for the differences between them. We also compare the modeled and measured ice loads from one well-documented storm that occurred at CRREL's freezing rain weather station. © 1998 Elsevier Science B.V.

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## 1. Introduction

There are many models that use meteorological data to calculate the amount of ice accreted on conductors and wires of overhead lines in freezing rain storms. The models differ in the level of detail in modeling the physical process of ice accretion, in the empirical data they use, and in the weather parameters that they require as input. Not surprisingly, they typically determine different ice loads for the same weather conditions. Although model verification has been attempted (Yip and Mitten, 1991), there is little high-quality, concurrent and collocated ice load and meteorological information to provide definitive tests for the models. Thus, in using ice accretion models to determine design ice loads for structures, one must choose a model without knowing either how representative the assumptions and empirical observations incorporated in the model are or how well the model represents reality.

* Corresponding author. Fax: +1-603-646-4644; e-mail: kjones@crrel.usace.army.mil


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In this paper, I develop a simple freezing rain ice accretion model, discuss the underlying assumptions and empirical observations, and show the dependence of the ice accretion rate on the weather parameters. I compare this simple model to four detailed ice accretion models and discuss the differences between them. Finally, I present a back-of-the-envelope formulation for the simple model.

## 2. Simple model

First, consider rain falling with no wind. The rain drop trajectories are vertical and perpendicular to the horizontal ground. The same depth of rain falls on a narrow sidewalk and a nearby wide highway. If the weather is cold enough and the highway and sidewalk are flat, so that the water does not pool or run off, the rainwater freezes to form a uniform layer of ice that is the same thickness on both structures. If the density $\rho_i$ of this ice is 0.9 g/cm$^3$, a 10-mm rainfall results in a uniform 11-mm thick ice layer. The mass of ice on a 100-m length of the highway is substantially greater than the mass of ice on a 100-m length of the sidewalk, but the ice thickness is the same on both.

Now extend this same argument to cylinders. Consider long circular cylinders of various diameters suspended horizontally above the ground in this same windless rainstorm. The 10 mm of rain that falls on the sidewalk and highway also falls on each of these cylinders. If all the impinging water freezes, and it freezes in a uniform radial accretion, then this 10 mm of rain is spread uniformly as ice over the surface of the cylinders. Because the perimeter is a factor of $\pi$ larger than the cylinder diameter, the uniform radial ice thickness $R_{eq}$ on each horizontal cylinder is:

$$R_{eq} = 10\frac{\rho_0}{\rho_i}\frac{1}{\pi} = 3.5\,\text{mm}, \tag{1}$$

where $\rho_0 = 1.0$ g/cm$^3$ is the density of water. As long as the ice accretes uniformly around the cylinder, the cylinder cross section remains circular. Therefore, the ratio of the diameter of each iced cylinder to the perimeter of its cross section remains $1/\pi = 0.32$ throughout the freezing rain storm, and the ice thicknesses on the cylinders are independent of their diameters. In general, for cylinders that do not have circular cross sections, such as angles, tees and rectangular tubing, the uniform ice thickness is proportional to the ratio of the dimension of the cylinder cross section intercepting the rain to the perimeter of the cross section (Jones, 1996).

Typically, there is wind during freezing rain storms, so we must also include the flux of windblown rain perpendicular to a vertical surface in the simple model. Best (1949) related liquid water content to precipitation rate, $W = 0.067P^{0.846}$, where $P$ is the precipitation rate in mm/h and $W$ is the liquid water content of the rain-filled air in g/m$^3$. Then, the flux of water perpendicular to a vertical surface is $VW$ (g/m$^2$ s), where $V$ is the wind speed in m/s. The water flux $w$ through a surface normal to the drop trajectories is obtained by converting to a consistent set of units and adding vectorially the contributions from windblown rain and falling rain $P\rho_o/10$ (g/cm$^2$ h):

$$w = \left[(0.1P\rho_0)^2 + (0.36VW)^2\right]^{1/2}\ \text{g/cm}^2\,\text{h}. \tag{2}$$

The uniform radial ice thickness on a circular cylinder is then:

$$R_{eq} = \frac{N}{\rho_i \pi} \left[ (P\rho_0)^2 + (3.6VW)^2 \right]^{1/2} \text{ mm}, \tag{3}$$

where $N$ is the number of hours of freezing rain with precipitation rate $P$ (mm/h) and wind speed $V$ (m/s).

During a storm, the precipitation rate and wind speed, which vary in time, are typically measured hourly at weather stations, so Eq. (3) can be written more generally as:

$$R_{eq} = \frac{1}{\rho_i \pi} \sum_{j=1}^{N} \left[ (P_j \rho_0)^2 + (3.6 V_j W_j)^2 \right]^{1/2}, \tag{4}$$

where $P_j$, $W_j = 0.067 P_j^{0.846}$, and $V_j$ are the precipitation rate, liquid water content and wind speed, respectively, in the $j$th hour of the storm lasting $N$ hours. This equation shows that the uniform radial ice thickness in the simple model is independent of cylinder diameter and depends only on two meteorological parameters: precipitation rate and wind speed. The nature of this dependence is illustrated in Fig. 1 for precipitation rates up to 10 mm/h and wind speeds up to 14 m/s.

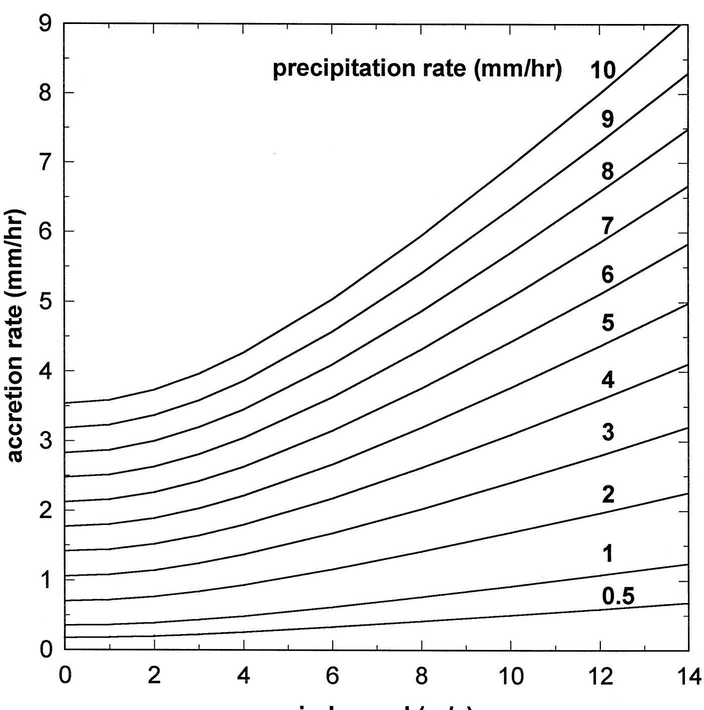


Fig. 1. Uniform radial ice accretion rate for the simple model as a function of precipitation rate ($0 < P < 10$ mm/h) and wind speed ($0 < V < 14$ m/s).

This simple ice accretion model for horizontal circular cylinders is based on three assumptions: (a) that the collision efficiency of the raindrops with the cylinder is 1 (Jones, 1996); (b) that all the rain water impinging on the cylinder sticks to the cylinder and freezes; and (c) that the ice accretes uniformly around the circumference of the cylinder. The first assumption is often made for the large drops (compared to cloud droplets) in freezing rain, and may be slightly conservative. In some freezing rain models (e.g., MRI, 1977), the collision efficiency is calculated incorrectly, ignoring the gravitational force on the raindrops. The second assumption, that all the impinging water freezes, is conservative. Detailed ice accretion models often include a heat-balance calculation to determine the fraction of the impinging precipitation that freezes. This calculation is based on numerous assumptions and empirical observations and requires additional meteorological data. The third assumption, that the accreted ice thickness is uniform, is a simplification of reality. Ice accretion shapes in freezing rain vary from a thin crescent on one side of the cylinder to heavily icicled shapes, an example of which is shown in Fig. 2. However, assuming a uniform radial accretion is consistent with the level of detail in this simple model, and is often assumed even in detailed models. The assumptions made in deriving the formula for glaze ice thickness presented in Goodwin et al. (1983) are similar to those in this model. However, the Goodwin formulation is given in terms of the fall speed of the rain drops. Thus, to implement that model, one would have to either assume a fall speed or incorporate a relationship between fall speed and precipitation rate in the model, using relationships in Best (1949), Marshall and Palmer (1948), or Wang and Pruppacher (1977), for example.

Fig. 2. Freezing rain ice accretion in Arkansas, February 1994 (photo Entergy).

The simple model is based on two empirical observations: (a) the density of the ice formed in freezing rain is 0.9 $g/cm^3$; and (b) the liquid water content in rain is related to the precipitation rate by Best's formula. I assumed a density for the accreted ice based on the typically clear ice accretions that are observed during freezing rain (Fig. 2). It is smaller than the density of pure bubble-free ice (0.917 $g/cm^3$). If the freezing rain is mixed with snow, the accreted ice density may be lower. I chose Best's formula for liquid water content rather than the Marshall–Palmer formula, which is used in some ice accretion models (e.g., Makkonen, 1996; Chaine and Castonguay, 1974). Best reviewed rain rate and liquid water content data from a number of researchers, including Marshall and Palmer (1948), and the relationship he recommended is the average of those he reviewed. Among these data, the Marshall–Palmer formula, $W = 0.72 P^{0.88}$, gives the highest liquid water content for a given precipitation rate. It is 5% higher than Best's at $P = 0.5$ mm/h and 15% higher at $P = 7$ mm/h. The effect on ice accretion rate of using the Marshall–Palmer $W$ instead of Best's $W$ varies with wind speed as well as precipitation rate. For relatively severe conditions with $P = 3$ mm/h and $V = 10$ m/s, using the Marshall–Palmer formula would give a 9% higher radial accretion rate than is calculated using Best's formula.

## 3. Comparison to detailed models

I compared the conservative ice loads determined by the simple model to ice loads calculated by four detailed freezing rain ice accretion models: the Chaine model (Chaine and Castonguay, 1974), the MRI model (MRI, 1977), the Makkonen freezing rain model (Makkonen, 1996), and the CRREL model (Jones, 1996). These four models are described in detail in the referenced papers and, except for the CRREL model, are summarized in Yip and Mitten (1991). I ran the models for all the freezing rain storms that occurred in Springfield, IL, between August 1948 and October 1993. I chose Springfield for this comparison because of its long computer-archived meteorological record with hourly weather and precipitation data and because of the relatively severe freezing rain storms in that region of the United States.

All the models require at least three weather parameters: precipitation rate, present weather code (which indicates whether the precipitation is freezing rain) and wind speed. The four detailed models also require air temperature data and require the user to specify the diameter of the cylinder on which the ice accretes. The CRREL model also uses hourly dew-point temperatures, atmospheric pressures and solar radiation fluxes. The MRI model, like the Goodwin model, requires the user to specify the fall speed of the raindrops. I modified the detailed models as necessary to treat the meteorological data in the same way so that the predicted ice loads are compared solely on the basis of the models' ice accretion algorithms. For example, ice was allowed to accrete only during freezing rain (the MRI model has algorithms to accrete rime ice and snow as well). The measured wind speeds were corrected, using the anemometer height history at Springfield and the 1/7 power law for exposure C (ASCE, 1993), to a constant height above ground (30 m).

I chose a fall speed for raindrops for the MRI model using the average precipitation rate during freezing rain at Springfield. In the 44 years with meteorological data at Springfield (weather data in 1992 was incomplete), there were 169 freezing rain storms with measurable precipitation. In these storms, the hourly precipitation rate varied from less than 0.5 up to 7 mm/h, with an average precipitation rate of about 1 mm/h. I used Best (1949) to determine the median volume drop diameter for $P = 1$ mm/h ($\sim 1$ mm) and Wang and Pruppacher (1977) to determine the terminal velocity for this size raindrop ($\sim 4$ m/s). I used this fall speed in running the MRI model with the Springfield data.

Fig. 3 compares the four detailed models with the simple model for a cylinder diameter of 10 mm, with a reference one-to-one line indicating perfect agreement on each plot. Because the simple model is conservative, freezing all the impinging precipitation, one might expect the detailed models to predict less ice and the plotted

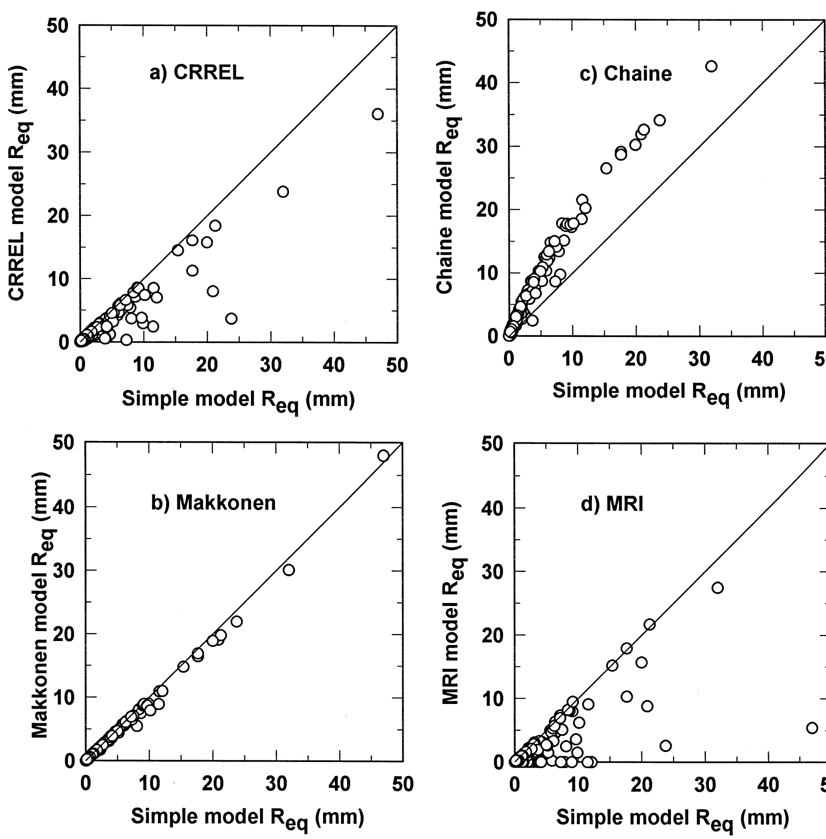


Fig. 3. Detailed ice accretion models compared to the simple model for a cylinder diameter of 10 mm using meteorological data from Springfield, IL: (a) CRREL model, (b) Makkonen model, (c) Chaine model, (d) MRI model.

points to fall below the one-to-one line. So, it is not surprising that the CRREL and MRI model results (Fig. 3a,d) both have the simple model results as an upper bound with many smaller ice thicknesses. On the other hand, the Makkonen model (Fig. 3b) agrees well with the simple model, and the Chaine model (Fig. 3c) predicts almost twice as much ice as the simple model for $R_{eq}$ smaller than about 10 mm, and about 10 mm more ice than the simple model for $R_{eq}$ larger than 10 mm.

The simple model predicts no dependence of the uniform radial ice thickness on cylinder diameter because it assumes that all the impinging precipitation freezes, and that it freezes with a uniform thickness. I checked this result in the detailed models by comparing the uniform radial ice thickness on the 10-mm-diameter cylinder with that on a 100-mm-diameter cylinder (Fig. 4). While the Chaine model shows significantly smaller ice thicknesses on the larger cylinder than on the smaller, the other three models show almost no dependence of ice thickness on cylinder diameter, even though the calculation of the freezing fraction in these models depends on the cylinder diameter and

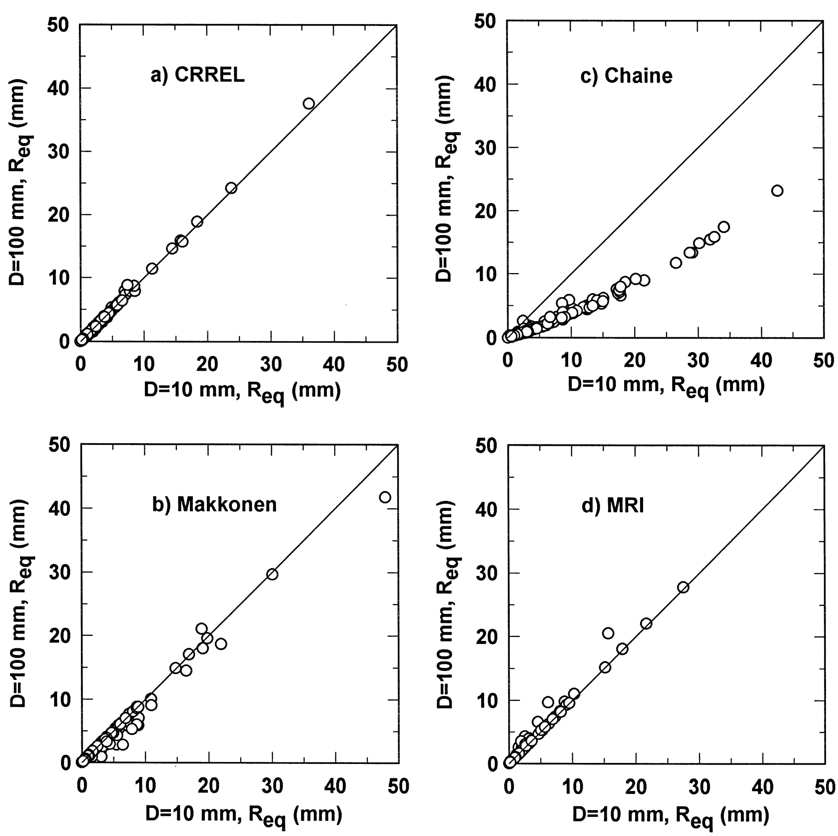


Fig. 4. Uniform radial ice thicknesses on 10-mm-diameter cylinder compared to 100-mm-diameter cylinder, using meteorological data at Springfield, IL: (a) CRREL model, (b) Makkonen model, (c) Chaine model, (d) MRI model.

even though the icicles, as well as the cylinder itself, in the Makkonen model directly accrete freezing rain.

While there are many differences in the formulations of the detailed models compared to the simple model, the differences, or lack thereof, between the detailed model results and the simple model results can be attributed to one or two aspects of each model.

The CRREL model ice thicknesses are typically less than the conservative ice thicknesses from the simple model. The differences are greater at high precipitation rates, air temperatures near freezing and relative humidities near 100%. Under these conditions icicles form in the CRREL model, but some of the rain still drips off, without freezing either on the cylinder or as an icicle. The uniform radial ice thickness, which includes icicles, does not depend on cylinder diameter in this model because model icicles form in many of the storms and their growth is independent of the cylinder Reynolds number, and thus diameter.

The Makkonen model agrees very well with the simple model. Almost all the impinging rain is frozen directly on the cylinder in this model with relatively less icicle mass than the CRREL model. This conservatism in the Makkonen model results primarily from two aspects of the model: (a) the accretion algorithm assumes that the ice is spongy, that is, that unfrozen water is incorporated in the ice that freezes to the cylinder; (b) the heat transfer coefficient used in calculating the freezing fraction is based on the Nusselt number averaged only over the windward half of a cylinder (Makkonen, 1985). This is appropriate in calculating the heat balance for cloud droplets impinging and freezing on the windward side of a cylinder in the usually windy conditions associated with in-cloud icing. In contrast, in the lower wind speeds (typically less than 10 m/s in the midwestern United States) that accompany freezing rain, drops of rain impinge on the top of the cylinder and freeze as they flow over its surface. These are the most significant factors in the good agreement between the Makkonen model and the conservative simple model.

The Chaine model ice thicknesses are significantly different from those determined by the simple model. While it is widely believed that the data on which the Chaine model is based were measured outdoors in freezing rain, that is not true. The model is based on wind-tunnel experiments to simulate sea spray icing (Stallabrass and Hearty, 1967) on horizontal and vertical cylinders, 38 mm in diameter and larger. In using the measured ice loads from these experiments to determine the correction factor that dominates their freezing rain model, Chaine and Castonguay (1974) made some rather odd choices. They used the ice thicknesses measured on each cylinder at approximately $\pm 45°$ around the cylinder from the stagnation point as if they were the accumulated ice thicknesses on horizontal and vertical plates (Yip, personal communication). The correction factors they calculated were then extrapolated to obtain correction factors for diameters as small as 6 mm. Thus, the Chaine model results that show greater ice thicknesses than the simple model for a 10-mm-diameter cylinder and a strong dependence of ice thickness on diameter are not credible.

With a specified 4 m/s fall speed for the raindrops, the MRI model typically freezes less ice than the CRREL model. The heat balance calculation in this model uses heat and mass transfer formulations for the cylinder that were developed for vapor deposition on

Table 1
Freezing rain storm, February 28, 1995, Hanover, NH

| Hour | Wind speed (m/s) | Weather | Air temperature (°C) | Dew point temperature (°C) | Precipitation (mm/h) |
|---|---|---|---|---|---|
| 0500 | 0.4 | S | −6.6 | −7.0 | 0.41 |
| 0600 | 0.0 | Z | −6.8 | −7.0 | 0.62 |
| 0700 | 0.7 | Z | −5.9 | −6.1 | 0.89 |
| 0800 | 1.0 | Z | −4.7 | −5.0 | 0.93 |
| 0900 | 1.2 | Z | −4.1 | −4.4 | 0.78 |
| 1000 | 1.5 | Z | −4.0 | −4.3 | 0.76 |
| 1100 | 1.3 | Z | −3.1 | −3.4 | 0.49 |
| 1200 | 1.3 | Z | −2.1 | −2.4 | 0.81 |
| 1300 | 1.4 | Z | −2.0 | −2.1 | 0.20 |
| 1400 | 1.7 | | −1.6 | −1.7 | 0.00 |

ice crystals (Koenig, 1971) and assumes 100% relative humidity. Less ice accretes on the cylinder and water that does not freeze immediately is assumed to drip off, rather than being available to freeze as icicles.

A comparison of the models using meteorological data and the measured ice load on a 26.5-mm-diameter cylinder at CRREL's freezing rain station during a freezing rain storm on February 28, 1995, provides another test for these models. At the end of this 10-h storm, characterized by cold temperatures, low wind speeds and low precipitation rates (Table 1), the equivalent radial ice thickness corresponding to the measured ice mass on the cylinder was 2.0 mm. In these conditions, the impinging rain froze quickly on the top of the cylinder and one would expect results from the simple model to be close to the CRREL, Makkonen and MRI models. In fact, all four models determined a uniform radial ice thicknesses of 1.9 mm for this storm, while the Chaine model ice thickness was 4.0 mm.

## 4. Back-of-the-envelope formulation

The simple model can be simplified further by determining a linear fit to Best's liquid water content formula, for the low precipitation rates associated with freezing rain, and incorporating the values for $\pi$, $\rho_0$, and $\rho_i$. Eq. (4) then becomes:

$$R_{eq} \approx 0.35 \sum_{j=1}^{N} P_j \left[ 1 + \left( \frac{V_j}{5} \right)^2 \right]^{1/2}, \tag{5}$$

for $P_j$ in mm/h, $V_j$ in m/s and $R_{eq}$ in mm. This formulation shows that windblown rain becomes as important as falling rain when the wind speed reaches 5 m/s. It gives conservative estimates for the accreted ice thickness that are as good as or better than those calculated by the detailed ice accretion models and requires no more than paper and pencil for implementation. I would use this formulation of the simple model to get a conservative estimate of the expected ice load on a wire in a predicted freezing rain storm.

## 5. Discussion

While the best information on accreted ice loads from freezing rain is provided by measurements of the ice mass on the structure of interest, these measurements are seldom made. Instead, ice loads are often modeled using meteorological data. The simple model presented in this paper is easy to use, conservative yet reasonable, and explicitly shows the dependence of the uniform radial ice thickness on the meteorological parameters.

The development of the model shows that, at least to first order, the uniform radial ice thickness does not depend on the cylinder diameter in freezing rain. This independence comes from assuming that the shape of the cross section of the cylinder plus accreted ice remains the same throughout the freezing rain storm and that all the impinging precipitation freezes. In contrast, if these same assumptions are made in modeling in cloud icing, the equivalent uniform radial ice thickness, using $\rho_i = 0.9$ g/cm$^3$, is not independent of cylinder diameter, because both the collision efficiency of the cloud droplets with the cylinder and the density of the accreted ice decrease as the diameter of the cylinder increases.

In reality, even in freezing rain, there may be an effect of cylinder diameter on the uniform radial ice thickness that is related to the actual shape of the ice on the cylinder. It is likely that this effect varies and depends on the meteorological conditions during each freezing rain storm.

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