Thoughts on extension of FV3GFS to the Whole Atmosphere

Extension of enthalpy and
Deep Atmospheric Dynamics

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Suggestions

- Follow what we have done on WAMGSM
- Do incremental implementation without changing too much on FV3 if possible
  - Consider generalized gas constituents Ri & Cpi
  - Extend vertical height with similar physics changes as done in WAMGSM
  - Add deep atmosphere dynamics (DAD) of r, g, and cosine components of Coriolis force

Step 1: Consider Ri and Cpi

- WAMGSM dynamics use enthalpy CpT
- FV3 uses virtual potential temperature
- Let's discuss from very beginning and possibly provide same idea of generalized gas constituents with Ri, and Cpi to form virtual potential temperature for FV3

Ideal gas law

Multi gas constituents can exist under a common temperature

$$p_i = \rho_i R_i T \qquad p = \rho R T$$

$$p = \sum_{i=0}^{n} p_i \qquad \rho = \sum_{i=0}^{n} \rho_i$$

Base air with n gas tracers

$$p = \sum_{i=0}^{n} p_i = \left(\sum_{i=0}^{n} \rho_i R_i \right) T = \rho \left(\sum_{i=0}^{n} \frac{\rho_i R_i}{\rho} \right) T = \rho R T$$

$$q_i = \frac{\rho_i}{\rho} \qquad R = \sum_{i=0}^{n} \frac{\rho_i}{\rho} R_i = \sum_{i=0}^{n} q_i R_i \qquad Cp = \sum_{i=0}^{n} q_i Cp_i$$

Gas constituent, in form of specific value
Non-gas tracer, in form of mixing ratio

Ideal gas law continue

Virtual temperature can be given as

$$p = \rho RT = \rho R_d T_v \qquad R_d = R_0 \qquad T_v = \frac{R}{R_d} T$$

$$R = \left(1 - \sum_{i=1}^{n} q_i\right) R_d + \sum_{i=1}^{n} q_i R_i \qquad \frac{R}{R_d} = \left(1 - \sum_{i=1}^{n} q_i\right) + \sum_{i=1}^{n} q_i \frac{R_i}{R_d} = 1 - \sum_{i=1}^{n} \left(1 - \frac{R_i}{R_d}\right) q_i$$

However, Rd here may not be dry air, but the base air as following

In GSM, we have gas and non-gas tracers as

	N2+O2+…	vapor	O3	water
Ri	286.05	461.50	173.22	0.0
Cpi	1004.6	1846.0	820.24	0.0

In WAM

	N2+…	vapor	O3	water	O	O2
Ri	296.80	461.50	173.22	0	519.67	259.84
Cpi	1039.6	1846.0	820.24	0	1299.2	918.10

Next we consider thermodynamic eqn from internal energy as

$$\frac{dC_P T}{dt} - \frac{RT}{p}\frac{dp}{dt} = F_T$$

The convenient way to solve this equation while Cp is not constant is by enthalpy h=CpT as a prognostic variable as in WAMGSM

However, GSM and FV3 solve system separately by adiabatic in dynamics and diabatic in model physics. For adiabatic sense, it will not be necessary to solve above equation with CpT together

In adiabatic $\frac{dq_i}{dt} = 0$ so $\frac{dCp}{dt} = \sum_{i=0}^{n}\left(Cp_i \frac{dq_i}{dt}\right) = 0$ $\frac{dR}{dt} = \sum_{i=0}^{n}\left(R_i \frac{dq_i}{dt}\right) = 0$

so

$$\frac{dC_P T}{dt} - \frac{RT}{p}\frac{dp}{dt} = C_P\frac{dT}{dt} + T\frac{dC_P}{dt} - \frac{RT}{p}\frac{dp}{dt} = C_P\frac{dT}{dt} - \frac{RT}{p}\frac{dp}{dt} = 0$$

Thus it is ok to solve $C_P\frac{dT}{dt} - \frac{RT}{p}\frac{dp}{dt} = 0$ But R and Cp are not constant

Under adiabatic, the thermodynamic eqn can be used to define potential temperature as

$$C_P \frac{dT}{dt} - \frac{RT}{p}\frac{dp}{dt} = 0 \quad => \quad \frac{d\ln T}{dt} - \frac{R}{C_P}\frac{d\ln p}{dt} = 0$$

While define $\theta = \frac{T}{\pi}$ where $\pi = \left(\frac{p}{p_0}\right)^{\frac{R}{Cp}}$ Note that $\frac{dq_i}{dt} = 0$ $\quad \frac{dR}{dt} = \frac{dCp}{dt} = 0$

$$\frac{d\ln\theta}{dt} = \frac{d\ln T}{dt} - \frac{d\ln\pi}{dt} = \frac{d\ln T}{dt} - \frac{R}{Cp}\frac{d\left(\ln p - \ln p_0\right)}{dt} = \frac{d\ln T}{dt} - \frac{R}{Cp}\frac{d\ln p}{dt} = 0 \quad => \quad \frac{d\theta}{dt} = 0$$

Potential temperature conservation under adiabatic with multi gases system

How about virtual potential temperature used in FV3?

We put $p = \rho RT = \rho R_d T_v$ into thermodynamic eqn we have

$$\frac{d\ln T}{dt} - \frac{R}{C_P}\frac{d\ln p}{dt} = \frac{d\ln\frac{R_d}{R}T_v}{dt} - \frac{R}{C_P}\frac{d\ln p}{dt} = \frac{d\ln T_v}{dt} - \frac{R}{C_P}\frac{d\ln p}{dt} = 0$$

so $\theta_v = \frac{T_v}{\pi}$ also conserved as defined $\frac{d\theta_v}{dt} = 0$

How to implement into FV3?

From nemsio or physics, we have T, qi, p

Use constant Ri and Cpi with qi to get R and Cp $R = \sum_{i=0}^{n} q_i R_i$ $Cp = \sum_{i=0}^{n} q_i Cp_i$

Use R and Cp to get pi $\pi = \left(\frac{p}{p_0}\right)^{\frac{R}{Cp}}$ $T_v = \frac{R}{R_d} T$ $\theta_v = \frac{T_v}{\pi}$

Use Rd, R to get Tv, with pi to get virtual potential temperature

Pass virtual potential temperature for FV dynamics $\frac{d\theta_v}{dt} = 0$

After dynamics get new qi and virtual potential T

Use constant Ri and Cpi with qi to get new R and Cp $R = \sum_{i=0}^{n} q_i R_i$ $Cp = \sum_{i=0}^{n} q_i Cp_i$

Use R and Cp to get new pi $\pi = \left(\frac{p}{p_0}\right)^{\frac{R}{Cp}}$ $T_v = \theta_v \pi$ $T = \frac{R_d}{R} T_v$

Use new pi, virtual potential T to get new Tv, then new T

Pass common new T, qi to physics or nemsio

Notes of step 1

- Gas tracers are treated in specific value, no-gas tracers are in mixing ratio. (from ideal gas law)
- Pi is defined by total R and Cp of all gas tracers, from thermodynamic eqn, no matter to define potential T or virtual potential T.
- Rd and Cpd to define pi is in the case of only dry air used in thermodynamic eqn, all other gas and non-gas tracers are mixing ratio; otherwise it is an approximation.
- Even though continuity eqn is for mass, since we use pressure form to represent it, we should use gas tracers in adiabatic dynamics, and leave non-gas tracers heating effect in diabatic physics.
- We provide a generalized eqn without approximation from ideal gas law and thermodynamic eqn under thermodynamic system.
- Implement these into FV3 dynamics first, model physics group can follow if accuracy is concerned.

Step 2: extend vertical + physics

- Using sigma-P coordinate to extend up to about 500 ~ 600 km with more layers, say 150.
- Use g(z) at some points as necessary.
- Following WAMGSM for WAM options with addition of some physics and diffusions…
  - Option with lsidea (diffusion or horizontal mixing)
  - Physics routines idea_phys, idea_* etc
- With step 1 and step 2 implemented in FV3, we come to the similar version as WAMGSM, and we have WAMFV3 even in nonhydrostatic system.
- During this stage, new physics can be added, and data assimilation can be tested.....

Step 3 deep atmosphere

- Try to implement the deep atmospheric dynamics (DAD) into FV3
- Modify the equation sets to have similar form as shallowness equation sets
- Using FV3 discretization and numerical techniques with additional terms and different facts in front of some terms like map factors in horizontal mapping.

Deep atmospheric equation in height & spherical coordinates

Momentum

$$\frac{du}{dt} - \frac{uv\tan\phi}{r} + \frac{uw}{r} - (2\Omega\sin\phi)v + (2\Omega\cos\phi)w + \frac{1}{\rho}\frac{\partial p}{r\cos\phi\partial\lambda} = F_u$$

$$\frac{dv}{dt} + \frac{u^2\tan\phi}{r} + \frac{vw}{r} + (2\Omega\sin\phi)u + \frac{1}{\rho}\frac{\partial p}{r\partial\phi} = F_v$$

$$\frac{dw}{dt} - \frac{u^2+v^2}{r} - (2\Omega\cos\phi)u + \frac{1}{\rho}\frac{\partial p}{\partial r} + g = F_w$$

where

$$\frac{dA}{dt} = \frac{\partial A}{\partial t} + u\frac{\partial A}{r\cos\phi\partial\lambda} + v\frac{\partial A}{r\partial\phi} + w\frac{\partial A}{\partial r} \qquad r = a + z$$

$$u = r\cos\phi\frac{d\lambda}{dt}$$

$$v = r\frac{d\phi}{dt}$$

Gas law

$$p = \sum_n p_n = \left(\sum_n \rho_n R_n\right)T = \rho\left(\sum_n \frac{\rho_n R_n}{\rho}\right)T = \rho\left(\sum_n q_n R_n\right)T = \rho R T$$

Density

$$\frac{\partial\rho}{\partial t} + \frac{\partial\rho u}{r\cos\phi\partial\lambda} + \frac{\partial\rho v\cos\phi}{r\cos\phi\partial\phi} + \frac{\partial\rho r^2 w}{r^2\partial r} = F_\rho$$

Thermodynamics

$$\frac{dC_P T}{dt} - \frac{RT}{p}\frac{dp}{dt} = F_T$$

Since FV3 needs flux form etc, let's start from deep atmosphere continuity equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{r\cos\phi \partial \lambda} + \frac{\partial \rho v \cos\phi}{r\cos\phi \partial \phi} + \frac{\partial \rho r^2 w}{r^2 \partial r} = 0$$

where

$$u = r\cos\phi \frac{d\lambda}{dt} \; ; \quad v = r\frac{d\phi}{dt}; \quad w = \frac{dr}{dt}$$

Derive into generalized vertical coordinate, we have (Juang 2014, ON#477)

$$\frac{\partial \rho^*}{\partial t} + m^2 \frac{\partial \rho^* \frac{u^*}{r}}{\partial \lambda} + m^2 \frac{\partial \rho^* \frac{v^*}{r}}{\partial \varphi} + \frac{\partial \rho^* \dot{\zeta}}{\partial \zeta} = 0$$

where $\rho^* = \rho \frac{r^2}{a^2}\frac{\partial r}{\partial \zeta}$; $u^* = u\cos\phi$; $v^* = v\cos\phi$; $m = \frac{1}{\cos\phi}$

Follow the same definition traditionally by

$$\frac{\partial \breve{p}}{\partial \zeta} = -\rho^{*} g_0 = -\rho \frac{r^2}{a^2} \frac{\partial r}{\partial \zeta} g_0 \qquad \Rightarrow \qquad \frac{\partial \breve{p}}{\partial \breve{\Phi}} = -\rho \; ; \quad \breve{\Phi} = \frac{g_0\left(r^3 - a^3\right)}{3a^2}$$

so

$$\frac{\partial \frac{\partial \breve{p}}{\partial \zeta}}{\partial t} + m^2 \frac{\partial \frac{\partial \breve{p}}{\partial \zeta} \frac{u^*}{r}}{\partial \lambda} + m^2 \frac{\partial \frac{\partial \breve{p}}{\partial \zeta} \frac{v^*}{r}}{\partial \varphi} + \frac{\partial \frac{\partial \breve{p}}{\partial \zeta} \dot{\zeta}}{\partial \zeta} = 0$$

let

$$\breve{u} = \frac{a}{r} u^* = \frac{a}{r} u \cos\phi \; ; \qquad \breve{v} = \frac{a}{r} v^* = \frac{a}{r} v \cos\phi$$

The above continuity becomes the same form as shallow atmosphere eqn

$$\frac{\partial \frac{\partial \breve{p}}{\partial \zeta}}{\partial t} + m^2 \frac{\partial \frac{\partial \breve{p}}{\partial \zeta} \breve{u}}{a \partial \lambda} + m^2 \frac{\partial \frac{\partial \breve{p}}{\partial \zeta} \breve{v}}{a \partial \varphi} + \frac{\partial \frac{\partial \breve{p}}{\partial \zeta} \dot{\zeta}}{\partial \zeta} = 0$$

In FV3 vertical Lagrangian form

$$D_L \delta \breve{p} + \nabla \bullet \left(\breve{V} \delta \breve{p}\right) = 0$$

Next, let see advection w.r.t. scaled horizontal wind

The advection in generalized coordinate, can be easily to use scaled wind as

$$\frac{d()}{dt} = \frac{\partial()}{\partial t} + m^2 u^* \frac{\partial()}{r\partial\lambda} + m^2 v^* \frac{\partial()}{r\partial\varphi} + \dot{\zeta}\frac{\partial()}{\partial\zeta}$$
$$= \frac{\partial()}{\partial t} + m^2 \breve{u}\frac{\partial()}{a\partial\lambda} + m^2 \breve{v}\frac{\partial()}{a\partial\varphi} + \dot{\zeta}\frac{\partial\breve{p}}{\partial\zeta}\frac{\partial()}{\partial\breve{p}}$$

And combine virtual potential temperature conservation with continuity equation, we can have

$$\frac{\partial\theta_v}{\partial t} + m^2\breve{u}\frac{\partial\theta_v}{a\partial\lambda} + m^2\breve{v}\frac{\partial\theta_v}{a\partial\varphi} + \dot{\zeta}\frac{\partial\breve{p}}{\partial\zeta}\frac{\partial\theta_v}{\partial\breve{p}} = 0$$

&

$$\frac{\partial\frac{\partial\breve{p}}{\partial\zeta}}{\partial t} + m^2\frac{\partial\frac{\partial\breve{p}}{\partial\zeta}\breve{u}}{a\partial\lambda} + m^2\frac{\partial\frac{\partial\breve{p}}{\partial\zeta}\breve{v}}{a\partial\varphi} + \frac{\partial\frac{\partial\breve{p}}{\partial\zeta}\dot{\zeta}}{\partial\zeta} = 0$$

$$\frac{\partial\frac{\partial\breve{p}}{\partial\zeta}\theta_v}{\partial t} + m^2\frac{\partial\frac{\partial\breve{p}}{\partial\zeta}\breve{u}\theta_v}{a\partial\lambda} + m^2\frac{\partial\frac{\partial\breve{p}}{\partial\zeta}\breve{v}\theta_v}{a\partial\varphi} + \frac{\partial\frac{\partial\breve{p}}{\partial\zeta}\dot{\zeta}\theta_v}{\partial\zeta} = 0$$

In FV3 vertical Lagrangian, we have

$$D_L\delta\breve{p}\theta_v + \nabla\bullet\left(\breve{V}\delta\breve{p}\theta_v\right) = 0$$

Since horizontal wind is scaled, how about vertical wind

We can start from definition of

$$\frac{\partial \breve{p}}{\partial \zeta} = -\rho^* g_0 = -\rho \frac{r^2}{a^2}\frac{\partial r}{\partial \zeta}\, g_0 = -\rho \frac{\partial \breve{\Phi}}{\partial \zeta}$$

so $$\frac{\partial \breve{\Phi}}{\partial \zeta} = \frac{r^2}{a^2}\frac{\partial r}{\partial \zeta}\, g_0 = \frac{g_0}{3a^2}\frac{\partial r^3}{\partial \zeta}$$

The general solution can be $\breve{\Phi} = \dfrac{g_0\left(r^3 - a^3\right)}{3a^2} = \dfrac{g_0\left(r-a\right)\left(r^2 + ra + a^2\right)}{3a^2}$

thus the vertical motion w.r.t new geopotential height

$$\frac{d\breve{\Phi}}{dt} = \frac{g_0 r^2}{a^2}\frac{dr}{dt} = g_0 \frac{r^2}{a^2} w$$

In shallowness $\Phi = g_0\left(r - a\right)$ so $\dfrac{d\Phi}{dt} = g_0 \dfrac{dr}{dt} = g_0 w$

In analogy, we let $\dfrac{d\breve{\Phi}}{dt} = g_0 \breve{w}$ so we get $\breve{w} = \dfrac{r^2}{a^2} w$

With 3D scaled winds as

$$\breve{u} = \frac{a}{r} u^* ; \quad \breve{v} = \frac{a}{r} v^* \quad ; \quad \breve{w} = \frac{r^2}{a^2} w$$

Their time derivatives are

$$\frac{d\breve{u}}{dt} = \frac{a}{r}\frac{du^*}{dt} - \frac{au^*}{r^2}\frac{dr}{dt} = \frac{1}{\varepsilon}\frac{du^*}{dt} - \frac{\breve{u}\breve{w}}{\varepsilon^3 a}$$

$$\frac{d\breve{v}}{dt} = \frac{a}{r}\frac{dv^*}{dt} - \frac{av^*}{r^2}\frac{dr}{dt} = \frac{1}{\varepsilon}\frac{dv^*}{dt} - \frac{\breve{v}\breve{w}}{\varepsilon^3 a}$$

$$\frac{d\breve{w}}{dt} = \frac{r^2}{a^2}\frac{dw}{dt} + \frac{2r}{a^2} w^2 = \varepsilon^2 \frac{dw}{dt} + \frac{2\breve{w}^2}{a\varepsilon^3}$$

where $\varepsilon = \frac{r}{a}$

The DAD scaled momentum eqns in shallowness form become

$$\frac{d\breve{u}}{dt} = -2\frac{\breve{u}\breve{w}}{\varepsilon^3 a}\delta - f_c^* \frac{\breve{w}}{\varepsilon^3}\delta + f_s \breve{v} - \frac{1}{\varepsilon^2}\left(\frac{1}{\rho}\frac{\partial p}{a\partial\lambda} + \frac{\partial p}{\partial \breve{p}}\frac{\partial \breve{\Phi}}{a\partial\lambda}\right)$$

$$\frac{d\breve{v}}{dt} = -2\frac{\breve{v}\breve{w}}{\varepsilon^3 a}\delta - f_s \breve{u} - m^2\frac{\breve{s}^2}{a}\sin\phi - \frac{1}{\varepsilon^2}\left(\frac{1}{\rho}\frac{\partial p}{a\partial\varphi} + \frac{\partial p}{\partial \breve{p}}\frac{\partial \breve{\Phi}}{a\partial\varphi}\right)$$

$$\frac{d\breve{w}}{dt} = 2\frac{\breve{w}^2}{\varepsilon^3 a}\delta + m^2\varepsilon^3\frac{\breve{s}^2}{a}\delta + m^2\varepsilon^3 f_c^* \breve{u}\delta + g_0\left(\frac{\partial p}{\partial \breve{p}}\varepsilon^4 - 1\right)$$

All terms with $\varepsilon = r / a$ and $\delta = 1$ are additions to shallow atmosphere dynamics

while $\varepsilon = 1$ and $\delta = 0$ all scaled wind and eqns are back to shallowness

And it is possible to define hydrostatic pressure as $\frac{\partial \bar{p}}{\partial \breve{p}}\varepsilon^4 - 1 = 0$

Apply vertical Lagrangian, we have

$$D_L\breve{u} = -\frac{\breve{w}}{\varepsilon^3}\left(2\frac{\breve{u}}{a} - f_c^*\right)\delta + f_s\breve{v} - \frac{1}{\varepsilon^2}\left(\frac{1}{\rho}\frac{\partial p}{a\partial\lambda} + \frac{\partial p}{\partial\breve{p}}\frac{\partial\breve{\Phi}}{a\partial\lambda}\right)$$

$$D_L\breve{v} = -\frac{\breve{w}}{\varepsilon^3}\frac{2\breve{v}}{a}\delta - f_s\breve{u} - m^2\frac{\breve{s}^2}{a}\sin\phi - \frac{1}{\varepsilon^2}\left(\frac{1}{\rho}\frac{\partial p}{a\partial\varphi} + \frac{\partial p}{\partial\breve{p}}\frac{\partial\breve{\Phi}}{a\partial\varphi}\right)$$

$$D_L\delta\breve{p}\breve{w} = \left(2\frac{\breve{w}^2}{\varepsilon^3 a} + m^2\varepsilon^3\frac{\breve{s}^2}{a} + m^2\varepsilon^3 f_c^*\breve{u}\right)\delta\breve{p}\delta + g_0\delta\breve{p}\frac{\partial p'}{\partial\breve{p}}\varepsilon^4$$

Add two previous obtained

$$D_L\delta\breve{p} + \nabla\bullet\left(\breve{V}\delta\breve{p}\right) = 0$$

$$D_L\delta\breve{p}\theta_v + \nabla\bullet\left(\breve{V}\delta\breve{p}\theta_v\right) = 0$$

We can see some terms are added to momentum eqns with facts of epsilon and delta, which won't influence numerical techniques inside FV3 core

The proper form of kinetic energy in FV3 will be checked later.

How to implement?

From IO or physics components, we have u, v, w, T, p, qi, and Ps or dz

Use Ri, Cpi, and qi to obtain $R,\ Cp, \pi, T_v, \theta_v$

Use Ps to get dp, and density to get dphi then get r $\frac{\partial \breve{p}}{\partial \breve{\Phi}} = -\rho$

Or dz to get r

compute all scaled values $\breve{u}\ ;\ \breve{v}\ ;\ \breve{w}$

Pass to dynamics to get prognostic values at next time step

Use new scaled prognostic values and others $\breve{u}\ ;\ \breve{v}\ ;\ \breve{w}\ ; \delta\breve{\Phi}\ ; q_i\ ; \theta_v$

Convert to obtain $R,\ Cp, \pi, T, \theta$

Get r from dz

Transform scaled wind into real wind

Pass to physics or write components

Note to All

- Separate dynamic and physics makes it easy to deal thermodynamic eqn with multi-gas. We suggest non-gas tracers should be treated in model physics as mixing ratio due to no contribution to pressure, though they can contribute to mass.
- Use scaled wind and factor (resemble the horizontal mapping) makes DAD in shallow atmosphere form without further complication
- Convert prognostic variables in one place between dynamics and physics/IO components makes it easy to implement.
- However, works have to be done in dynamics, R and Cp have to update whenever qi are updated before any conversion among T, Tv, and (virtual) potential T.
- Also, the terms with epsilon and where/when to add delta terms have to take care in FV3 dynamics.
- We do implement DAD with backward compatibility, so when epsilon=1 and delta=0, all are back to original code.