Evaluation of NCAR's AutoNowCaster for Operational Application within the National Weather Service

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The AutoNowCaster (ANC) Team

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NCAR / RAL

Rita Roberts, Principal Investigator

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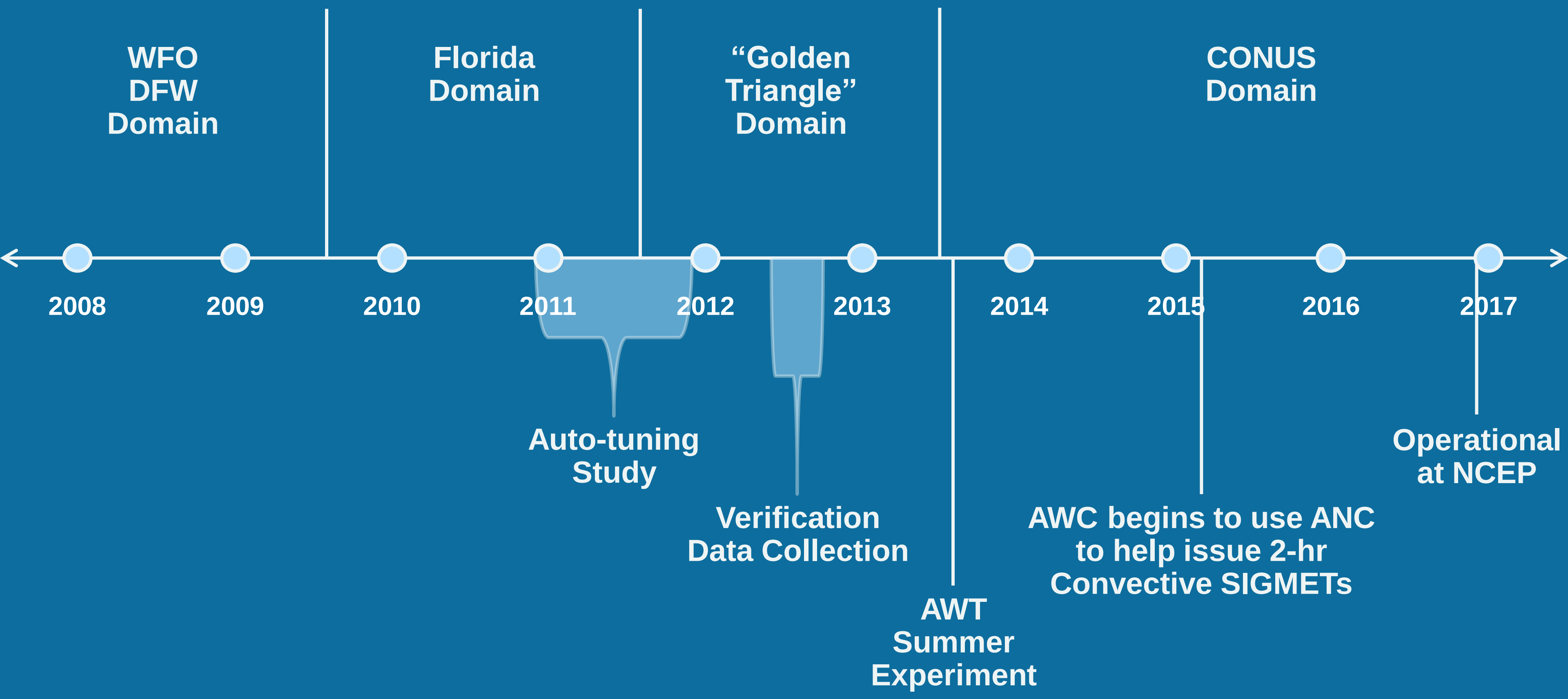
ANC: MDL Timeline



ANC: Overview

- ANC is designed to nowcast areas where storms are considered likely to form and/or be sustained, and vice-versa. How?

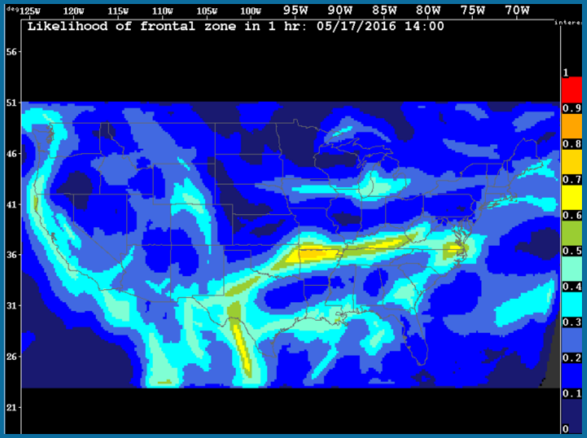
- ANC ingests NWP model output, GOES satellite data, surface METAR data, and NWS radiosonde data in order to analyze characteristic features of the atmosphere. The results of the analyses are 60-minute predictors which are converted into dimensionless likelihood fields.

- The likelihood fields have a dynamic range from -1 to 1, where increasing positive values correspond to an increasing likelihood of storm initiation and/or sustainment, and vice-versa.

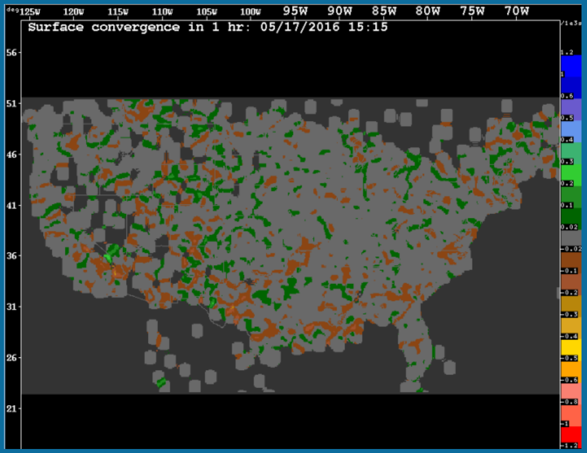
- The likelihood fields are weighted and summed to produce a 60-minute Convective Likelihood (CL) field.

- In essence, ANC attempts to analyze, weight, and sum various data in a manner analogous to that of a forecaster.

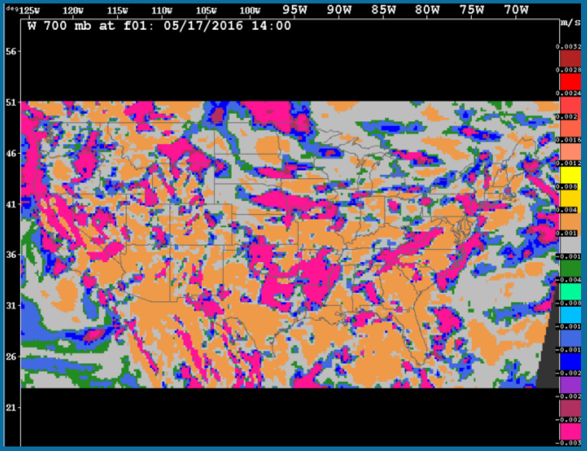
ANC Predictors: Dynamics



Likelihood of Frontal Zone in 1 hr

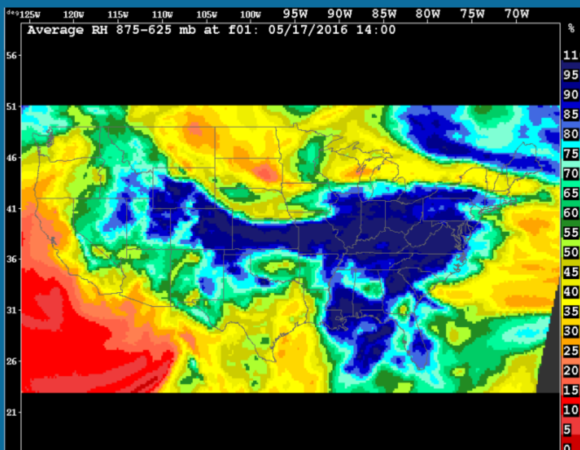


Surface Convergence in 1 hr

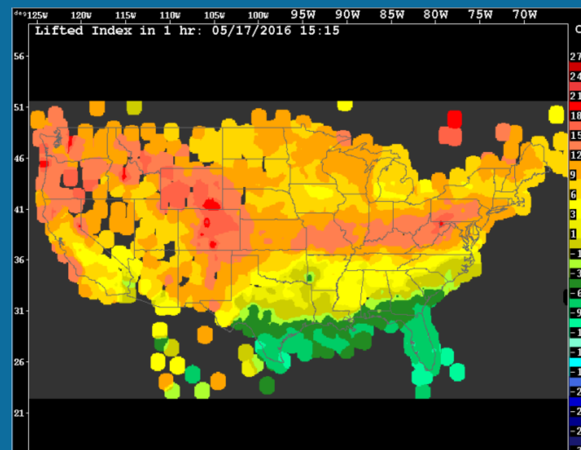


W 700 mb at f01

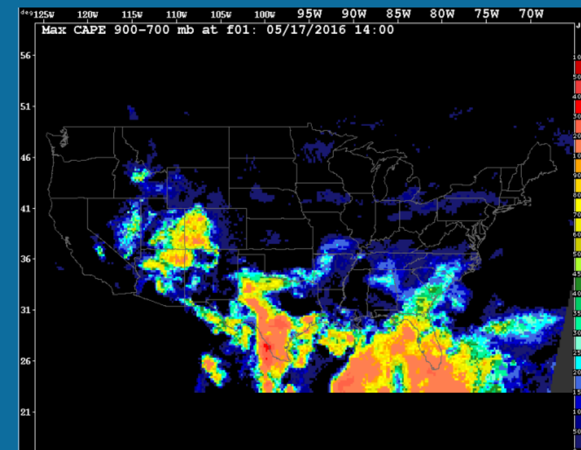
ANC Predictors: Thermodynamics



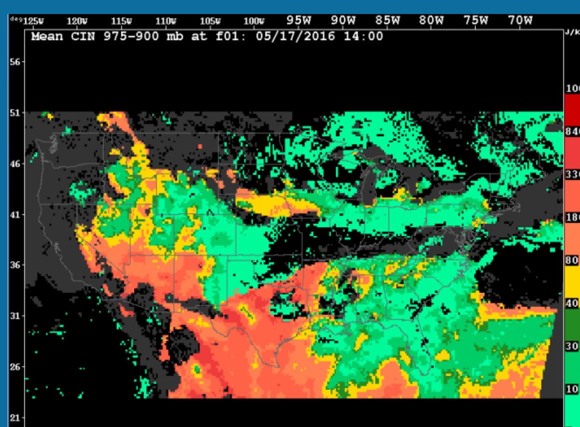
Average RH 875-625 mb at f01



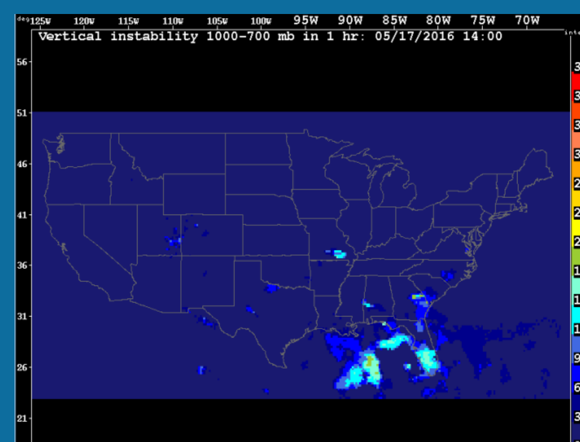
Lifted Index in 1 hr



Max CAPE 900-700 mb at f01

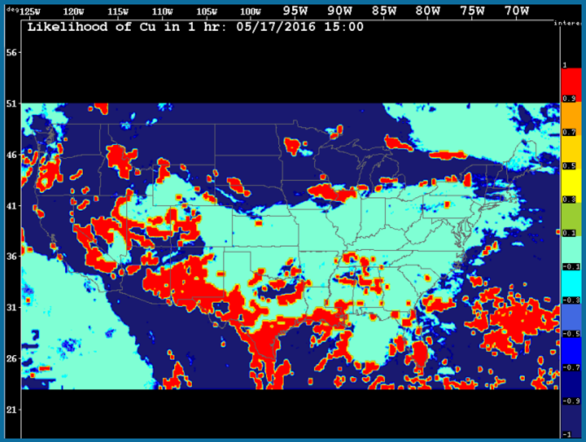


Mean CIN 975-900 mb at f01

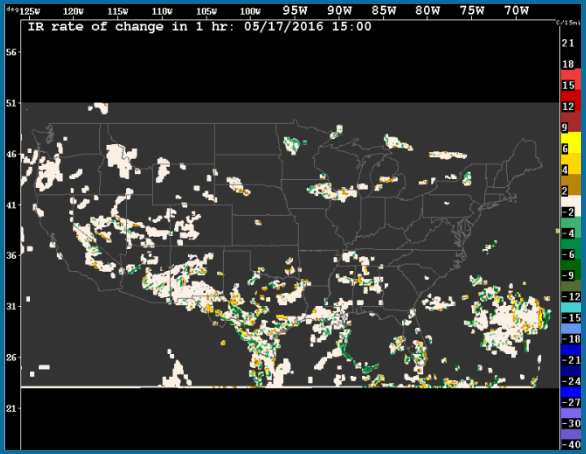


Vertical Instability 1000-700 mb in 1 hr

ANC Predictors: Clouds



Likelihood of Cu in 1 hr

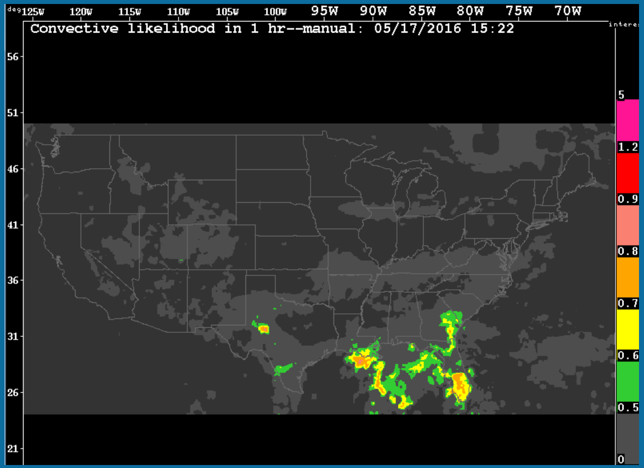


IR Rate of Change in 1 hr

ANC Predictand:
Convective Likelihood (CL) in 1 hr

(0.22 * F_1 (Likelihood of Frontal Zone in 1 hr)) +
(0.08 * F_2 (W 700 mb at f01)) +
(0.10 * F_3 (Surface Convergence in 1 hr)) +
(0.18 * F_4 (Average RH 875-625 mb at f01)) +
(0.20 * F_5 (Lifted Index in 1 hr)) +
(0.20 * F_6 (Max CAPE 900-700 mb at f01)) +
(0.12 * F_7 (Mean CIN 975-900 mb at f01)) +
(0.12 * F_8 (Vertical Instability 1000-700 mb in 1 hr)) +
(0.12 * F_9 (Likelihood of Cu in 1 hr)) +
(0.40 * F_{10} (Likelihood of Clear Sky in 1 hr)) +
(0.10 * F_{11} (IR Rate of Change in 1 hr))

=



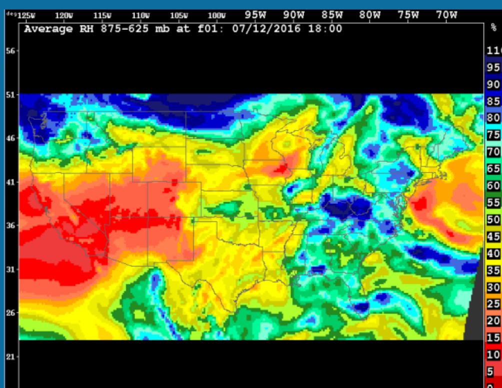
Convective Likelihood in 1 hr

$F_{\#}$ denotes a predictor-specific fuzzy function.

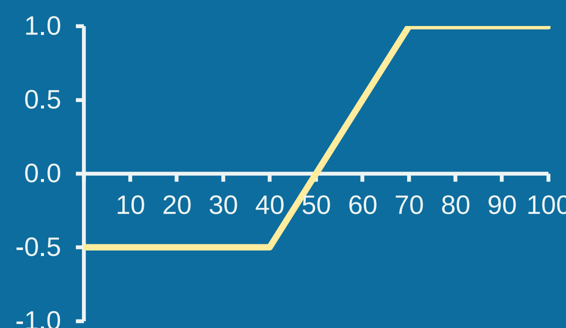
ANC: Predictor-Specific Fuzzy Functions

PREDICTOR

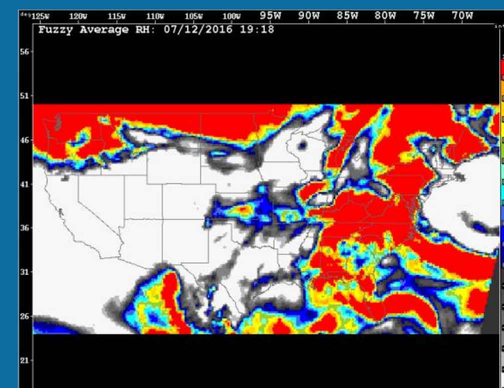
LIKELIHOOD FIELD



Average RH
875-625 mb at f01



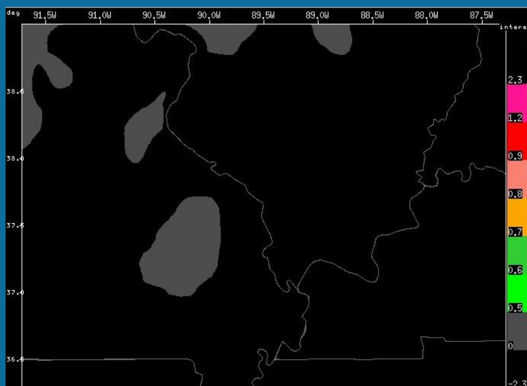
Fuzzy Function for Average RH
875-625 mb at f01



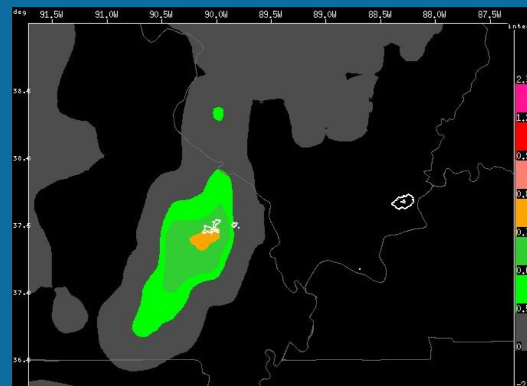
“Fuzzy” Average RH
875-625 mb at f01

See Appendix D for the graphs of ANC’s predictor-specific fuzzy functions.

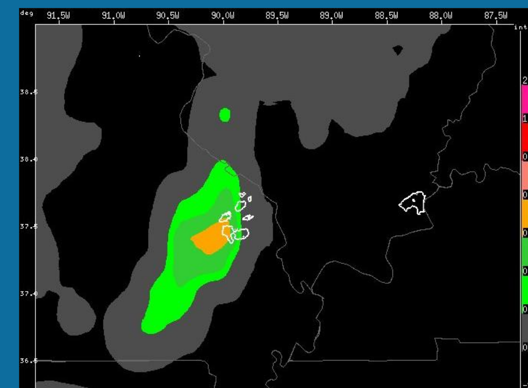
ANC: Example Time Series



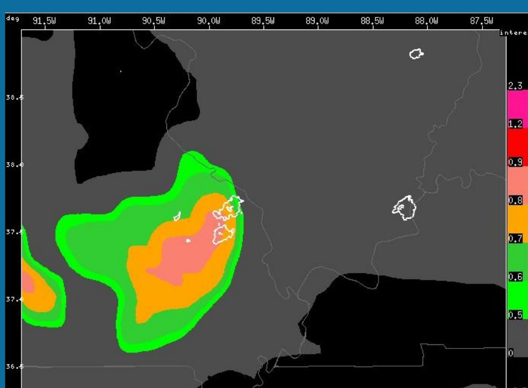
Valid time: 1741Z



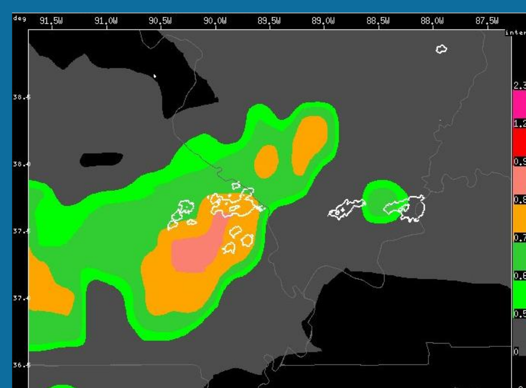
Valid time: 1838Z



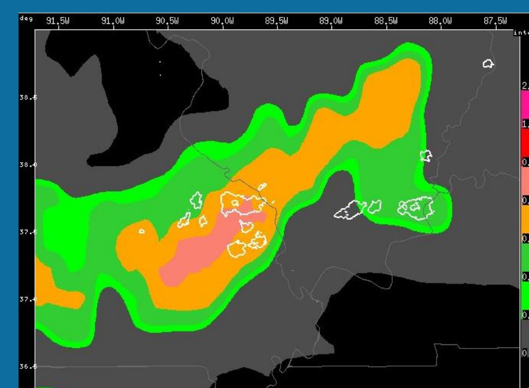
Valid time: 1853Z



Valid time: 1908Z



Valid time: 1929Z



Valid time: 1944Z

60-min nowcasts of CL on July 1, 2012 overlaid with 35 dBZ contours at the valid time

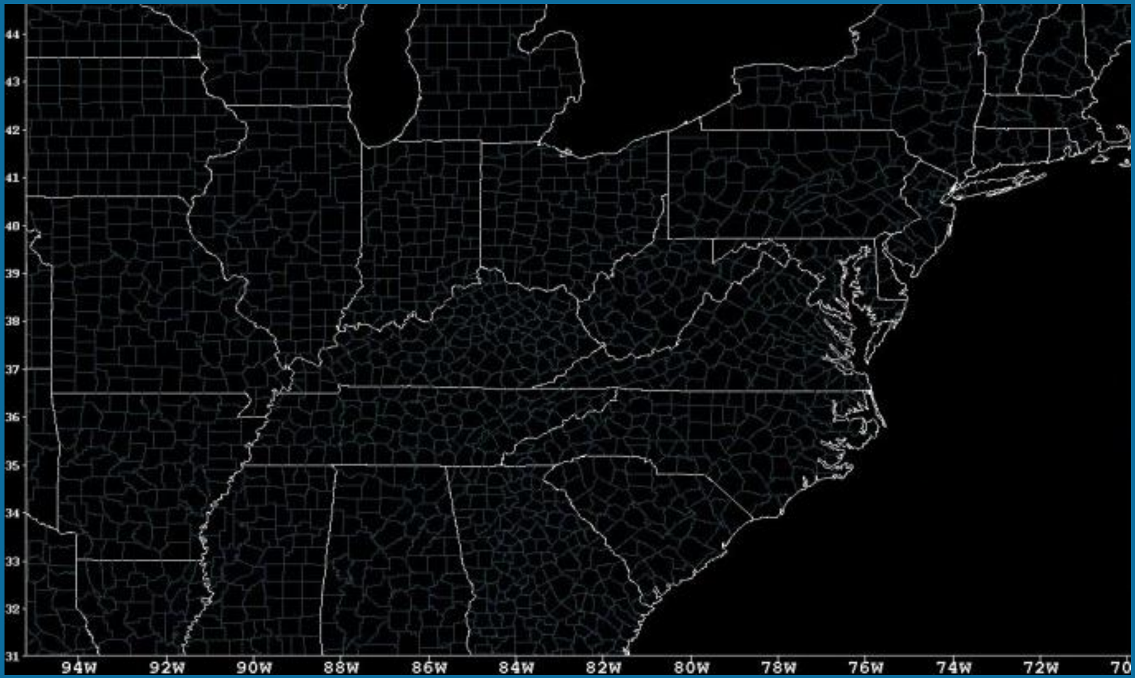
ANC Verification: Questions

- **What is the smallest spatial scale at which ANC's 60-minute nowcasts of CL can skillfully nowcast the general areas where *both* new storms may initiate *and* existing storms should be sustained, and to what values of CL does this apply?**

- **What is the smallest spatial scale at which ANC's 60-minute nowcasts of CL can skillfully nowcast the general areas *solely* where new storms may initiate, and to what values of CL does this apply?**

- **To what degree, if any, are ANC's 60-minute nowcasts of CL subject to temporal ambiguity?**

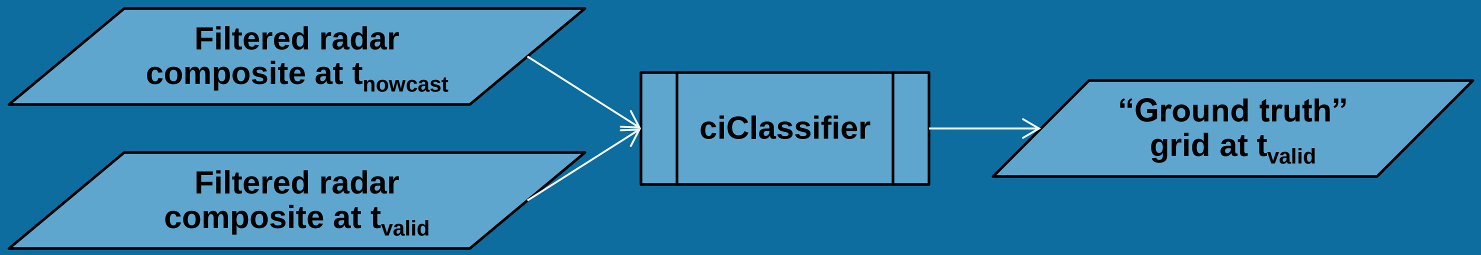
ANC Verification: Domain



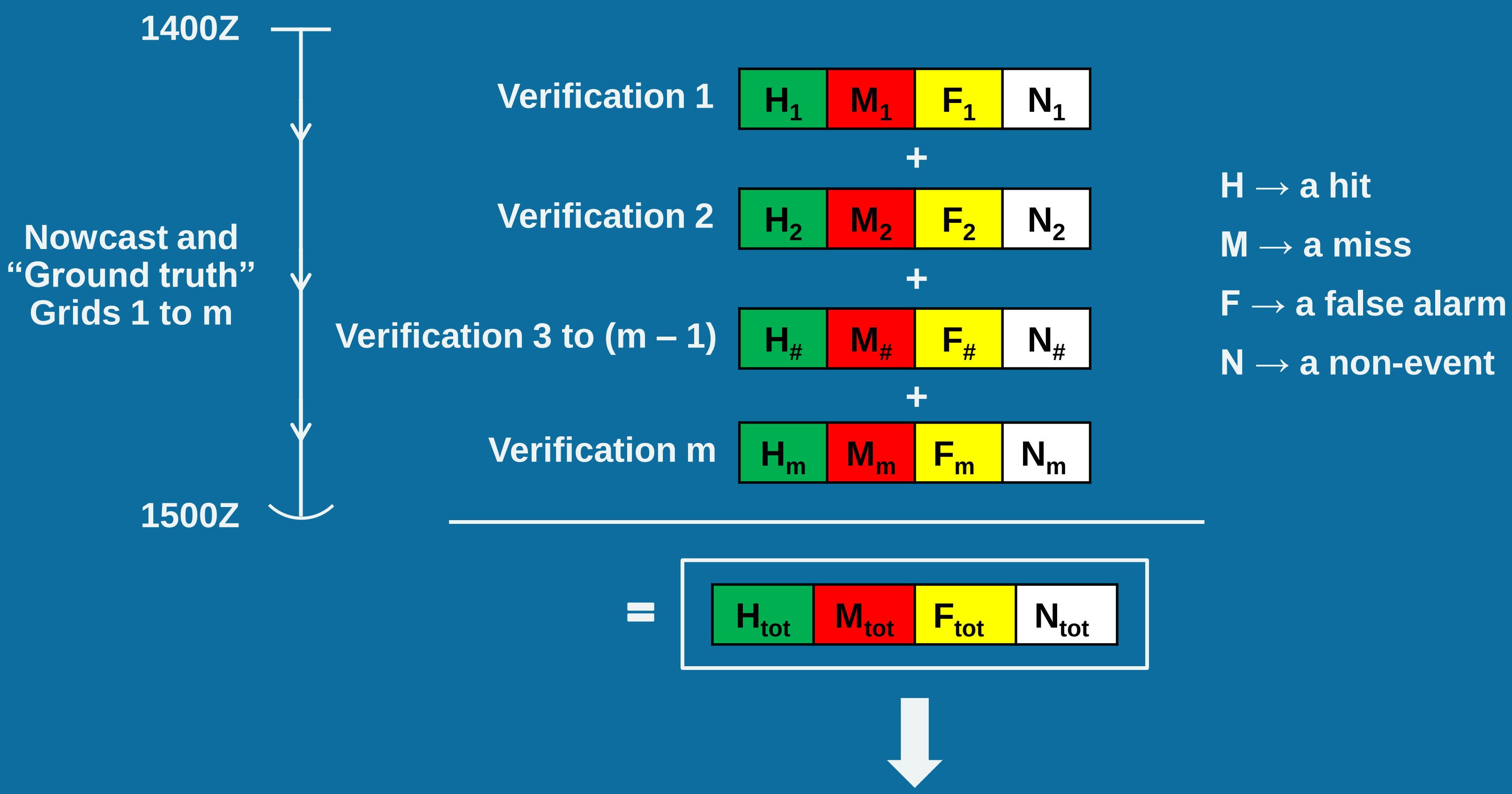
The "Golden Triangle" Domain: 31° to 45° N; 94° to 71° W; 0.02° x 0.02°

ANC Verification: Data

1. Nowcast grids of ANC's predictand: CL in 1 hr
   - 55 days from June 11 to September 30, 2012; 1400Z to 2359Z; ~10 nowcasts per hour
2. "Ground truth" grids derived from 3-D Cartesian WSR-88D reflectivity volumes
   - Reflectivity volume
     - ↳ Filtered for bright band data
       - ↳ Filtered for stratiform data
         - ↳ Composited between 2.5 and 4.5 km, inclusive



ANC Verification: Statistics



Calculate POD, FAR, Bias, CSI and HSS for the hour

ANC Verification: Spatial Neighborhoods

N = the neighborhood size parameter

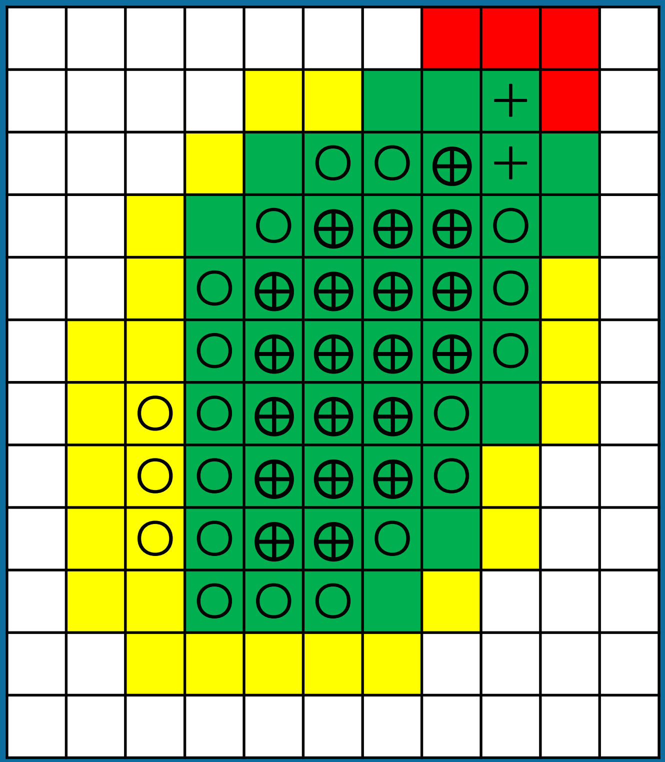
It yields a (2N + 1) by (2N + 1) neighborhood.

○ ⟶ an event was forecasted

+ ⟶ an event was observed

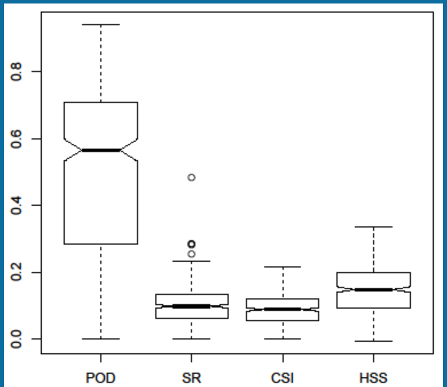
⊕ ⟶ an event was both forecasted and observed

- (green) → a hit
- (red) → a miss
- (yellow) → a false alarm
- (white) → a non-event



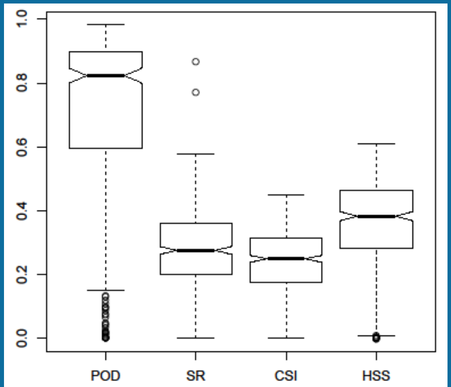
Example verification when N = 1, i.e., a 3 by 3 neighborhood

ANC Verification: Results (1)



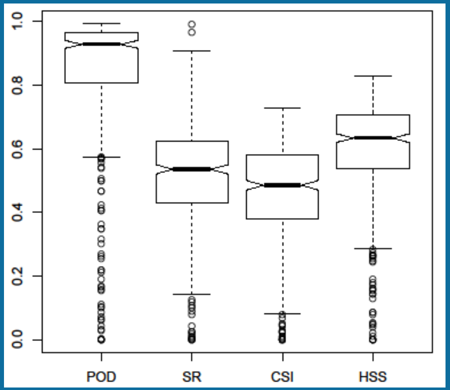
N = 2 (~10 km²)

Median CSI ~0.1



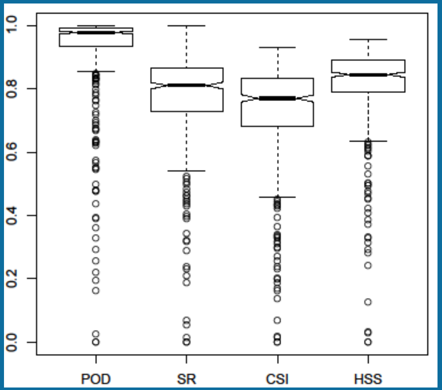
N = 6 (~25 km²)

Median CSI ~0.3



N = 12 (~50 km²)

Median CSI ~0.5

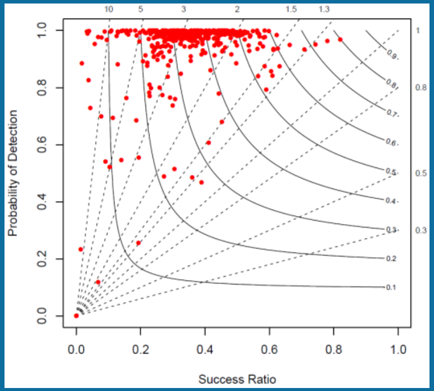


N = 24 (~100 km²)

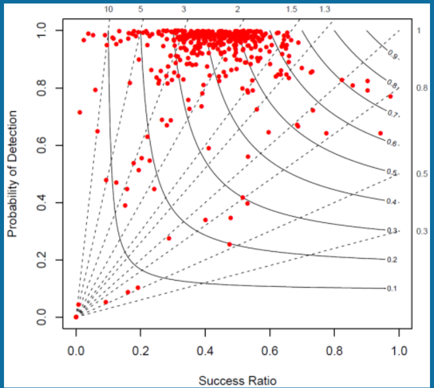
Median CSI ~0.8

Verification for N = 2 (~10 km²), 6 (~25 km²), 12 (~50 km²), and 24 (~100 km²); a forecast event consists of a nowcast grid point whose CL value ≥ 0.6; an observed event consists of a "ground truth" grid point classified *either* as storm initiation *or* as an ongoing storm

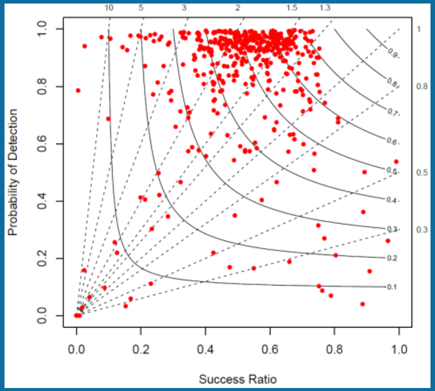
ANC Verification: Results (2)



CL ≥ 0.4



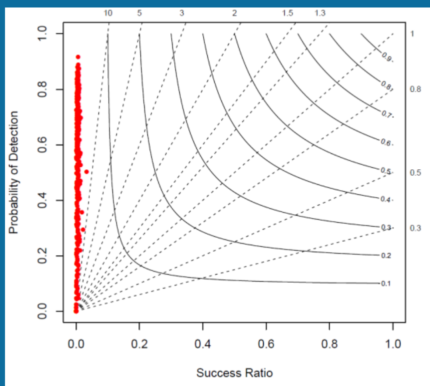
CL ≥ 0.5



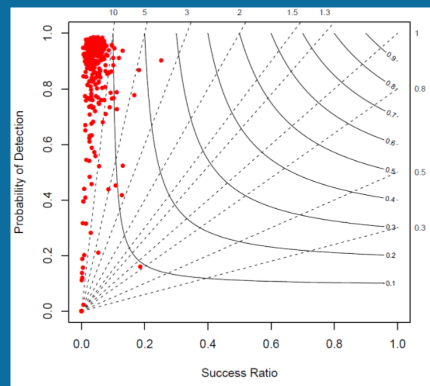
CL ≥ 0.6

Verification for N = 12 (~50 km^2); a forecast event consists of a nowcast grid point whose CL value ≥ 0.4, 0.5, and 0.6; an observed event consists of a "ground truth" grid point classified *either* as storm initiation *or* as an ongoing storm

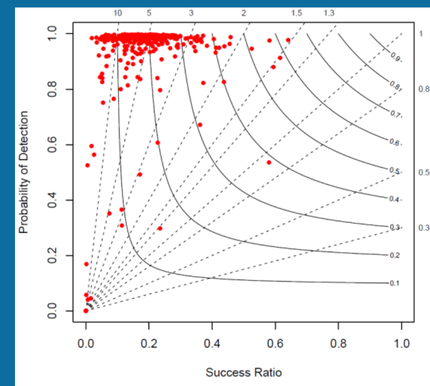
ANC Verification: Results (3)



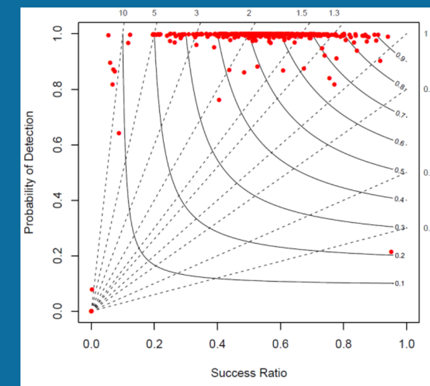
N = 2 (~10 km^2)



N = 6 (~25 km^2)



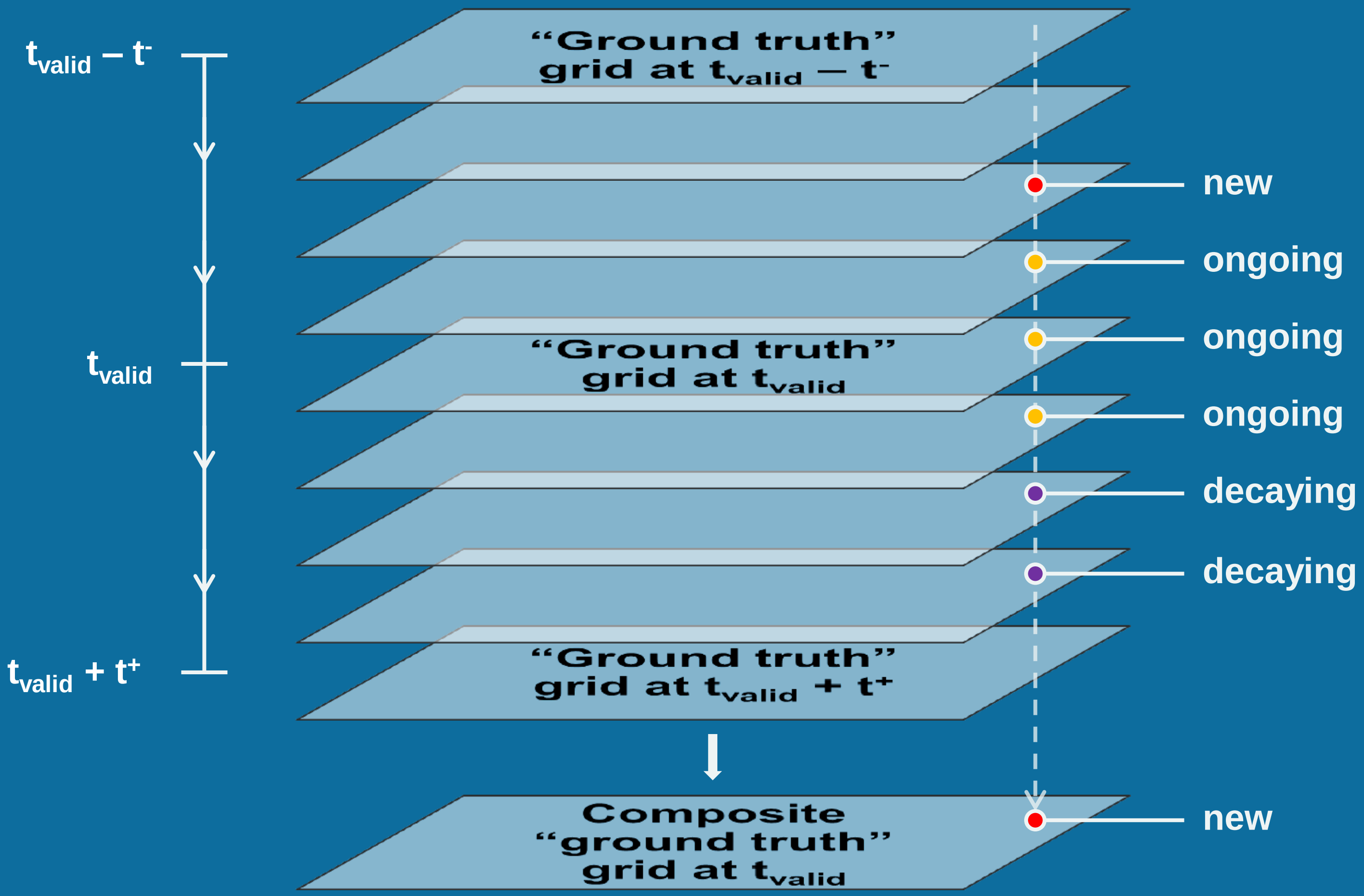
N = 12 (~50 km^2)



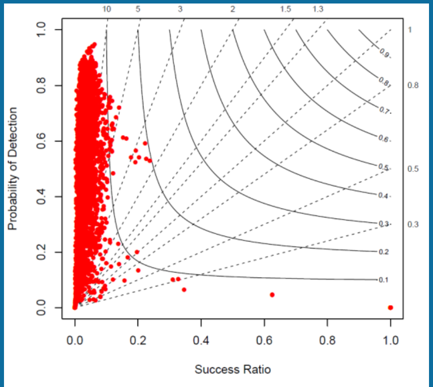
N = 24 (~100 km^2)

Verification for N = 2 (~10 km^2), 6 (~25 km^2), 12 (~50 km^2), and 24 (~100 km^2); a forecast event consists of a nowcast grid point whose CL value ≥ 0.7; an observed event consists of a “ground truth” grid point classified *solely* as storm initiation

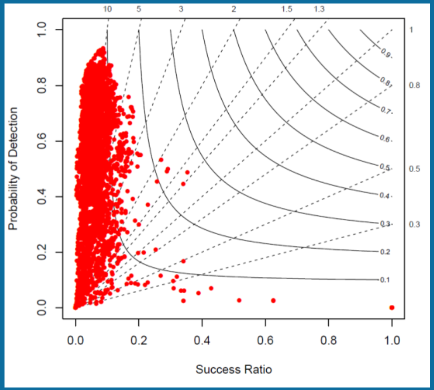
ANC Verification: Temporal Relaxation



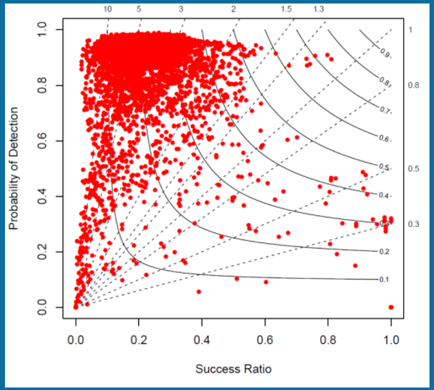
ANC Verification: Results (4)



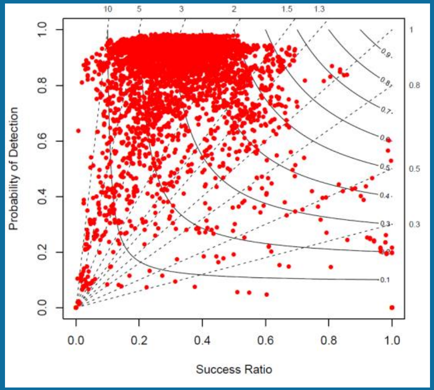
N = 2 (~10 km^2);
t^- and t^+ = 15 min



N = 2 (~10 km^2);
t^- and t^+ = 30 min



N = 6 (~25 km^2);
t^- and t^+ = 15 min



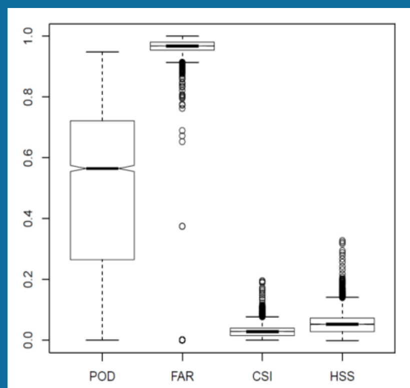
N = 6 (~25 km^2);
t^- and t^+ = 30 min

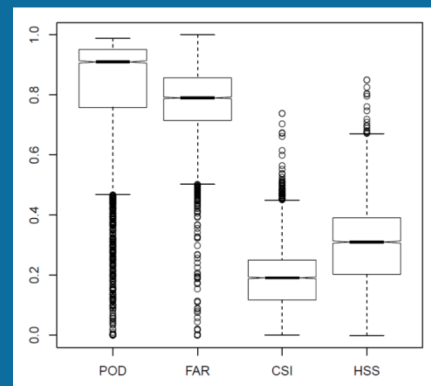
Verification for N = 2 (~10 km^2) and 6 (~25 km^2); t^- and t^+ = 15 and 30 min; a forecast event consists of a nowcast grid point whose CL value ≥ 0.7; an observed event consists of a "ground truth" grid point classified *solely* as storm initiation

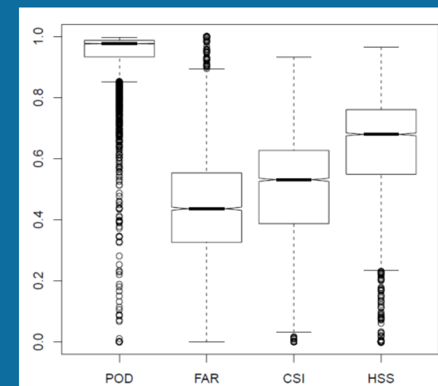
ANC Verification: Results (5)

N = 2 (~10 km^2) | N = 6 (~25 km^2) | N = 12 (~50 km^2)

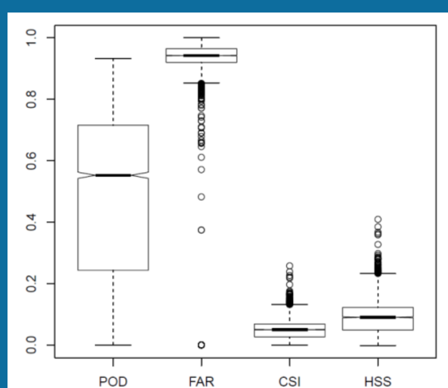
t^- and t^+ = 15 min

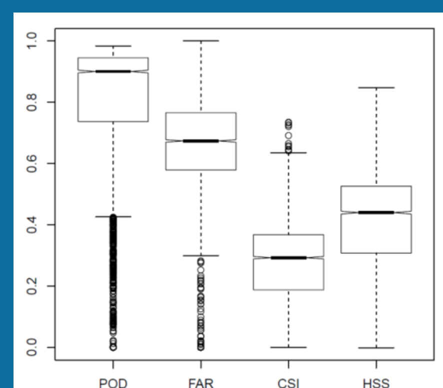


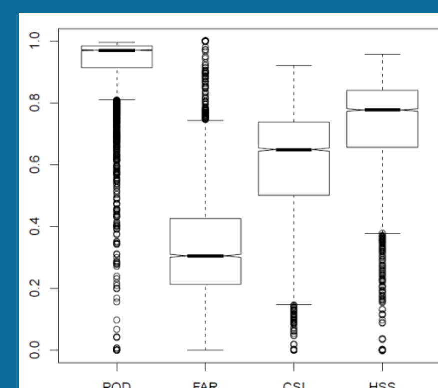




t^- and t^+ = 30 min







Verification for N = 2 (~10 km^2), 6 (~25 km^2), and 12 (~50 km^2); t^- and t^+ = 15 and 30 min; a forecast event consists of a nowcast grid point whose CL value ≥ 0.7; an observed event consists of a "ground truth" grid point classified *solely* as storm initiation

ANC Verification: Conclusions

- At a spatial scale of ~50 km and with no temporal relaxation, grid points with values ≥ 0.6 in ANC's 60-minute nowcasts of CL skillfully nowcast the general areas where both new storms may initiate and existing storms should be sustained.

- At a spatial scale of ~50 km and within 45 to 90 minutes from the nowcast issuance time, grid points with values ≥ 0.7 in ANC's 60-minute nowcasts of CL skillfully nowcast the general areas where new storms may initiate.

- ANC's 60-minute nowcasts of CL can best improve situational awareness when interpreted as guidance at a spatial scale of ~50 km and within a time frame anywhere between 45 and 90 minutes of the issuance times.

ANC: MDL Points of Contact

To express an interest in collaborative research and development with ANC, contact Dr. Stephan (Steve) Smith.

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For scientific questions about ANC, contact Dr. Mamoudou (Ama) Ba.

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For questions about the verification analysis, contact Dr. Lingyan Xin.

Email: lingyan.xin@noaa.gov

Telephone: (301) 427-9092

For technical questions about the ANC system, its configuration, etc., contact John Crockett.

Email: john.crockett@noaa.gov

Telephone: (301) 427-9469

ANC: Select Bibliography (1)

- Ba, M., L. Xin, J. Crockett, and S. Smith: Evaluation of NCAR's AutoNowCaster for Operational Application within the National Weather Service. *Wea. Forecasting*, **32**, 1477-1490. *DOI: 10.1175/WAF-D-16-0173.1*

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ANC: Select Bibliography (2)

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- Wolfson, M. M., B. E. Forman, R. G. Hallowell, and M. P. Moore, 1999: The growth and decay storm tracker. Preprints, *8th Conf. on Aviation, Range, and Aerospace Meteorolog*y, Dallas, TX, Amer. Meteor. Soc., 4.3. *https://www.ll.mit.edu/mission/aviation/publications/publication-files/ms-papers/Wolfson_1999_ARAM_MS-13335_WW-10100.pdf*

Appendix A: Creating the Likelihood of Frontal Zone in 1 hr

This field "is used to determine the orientation of large-scale frontal forcing." (Megenhardt et al., 2004) It "is designed to identify meso-synoptic (L = 100-1000 km) frontal structures based on . . . vorticity, horizontal divergence and horizontal gradients of θ_e. By itself, this field tends to identify horizontal scales that are significantly greater than those over which the organized convection occurs. However, it does provide important information on large-scale forcing that helps support the mesoscale organization of convection." (Megenhardt et al., 2004)

At each grid point, calculate the following:

((0.5 * F_1 (Convergence 1000 mb at f03)) +

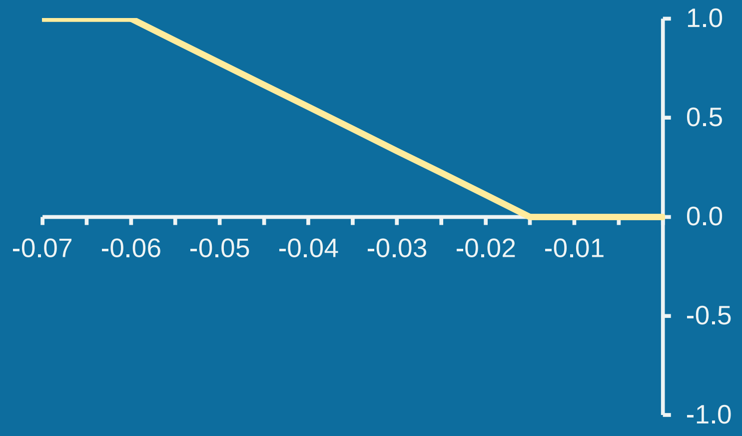
(1.0 * F_2 (Maximum gradient of θ_e 1000 mb at f03)) +

(0.5 * F_3 (Vorticity 1000 mb at f03))) / 2.0

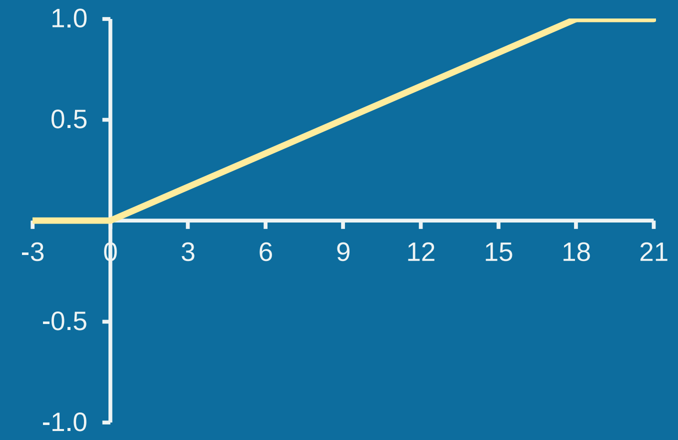
where $F_{\#}$ denotes the appropriate field-specific fuzzy function from the next two slides.

The above normalized, weighted summation is then input to an elliptical filter algorithm created by MIT Lincoln Labs. See Wolfson et al., 1999.

Convergence 1000 mb at f03



Maximum gradient of θ_e 1000 mb at f03



$$y = 1:\ \ x \le -0.06\ /10^3 s$$

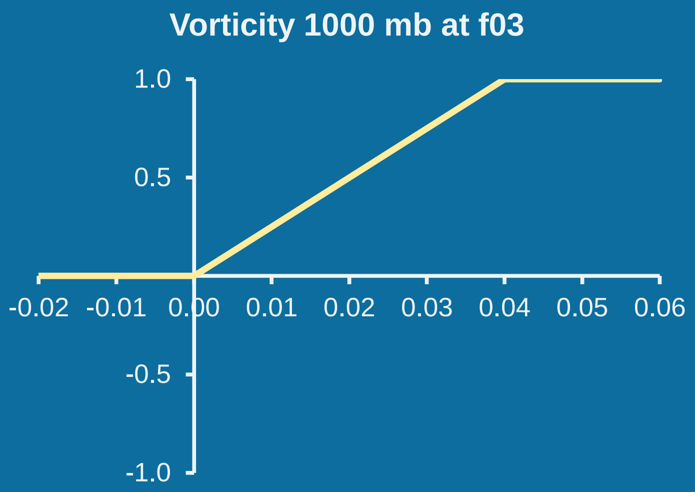
$$y = -22.\overline{2}x - 0.\overline{3}:\ \ -0.06\ /10^3 s < x < -0.015\ /10^3 s$$

$$y = 0:\ \ -0.015\ /10^3 s \le x$$

$$y = 0:\ \ x \le 0K$$

$$y = 0.0\overline{5}x:\ \ 0K < x < 18K$$

$$y = 1:\ \ 18K \le x$$



$$y = 0\text{:}\ \ x \leq 0\ /10^3 s$$

$$y = 25x\text{:}\ \ 0\ /10^3 s < x < 0.04\ /10^3 s$$

$$y = 1\text{:}\ \ 0.04\ /10^3 s \leq x$$

Appendix B: Creating the Vertical Instability 1000-700 mb in 1 hr

This field "is a two-dimensional . . . field that approximates the 'depth' of thermodynamic instability." It is based on the idea that "a single deep layer of instability, in most instances, is more conducive to deep convection than several shallow layers of instability that are separated by some distance in the vertical." (Megenhardt et al., 2004)

Step 1 of 7

For each level between 1000 and 700 mb, inclusive, estimate the next CAPE and CIN analysis fields.

Step 2 of 7

Calculate the f01 layer average relative humidity between 875 and 625 mb, inclusive.

Step 3 of 7

Calculate the f01 vertical derivative of horizontal θ_e advection from 875 to 675 mb.

Step 4 of 7

Calculate the f01 magnitude of vertical shear between 975 and 725 mb, inclusive.

Step 5 of 7

For each level between 1000 and 700 mb, inclusive, correct the estimate of the next CAPE analysis field using the following equation:

$$((1.0 * F_1 (\text{Estimate of CAPE}_{f00} \text{ at t + 1})) +$$
$$(0.357 * F_2 (\delta_{CAPE} \text{ Average RH 875-625 mb at f01})) +$$
$$(0.357 * F_3 (\delta_{CAPE}\ d\theta_e/dz \text{ 875-675 mb at f01})))$$

where $F_{\#}$ denotes the appropriate field-specific fuzzy function from the next three slides.

Step 6 of 7

For each level between 1000 and 700 mb, inclusive, correct the estimate of the next CIN analysis field using the following equation:

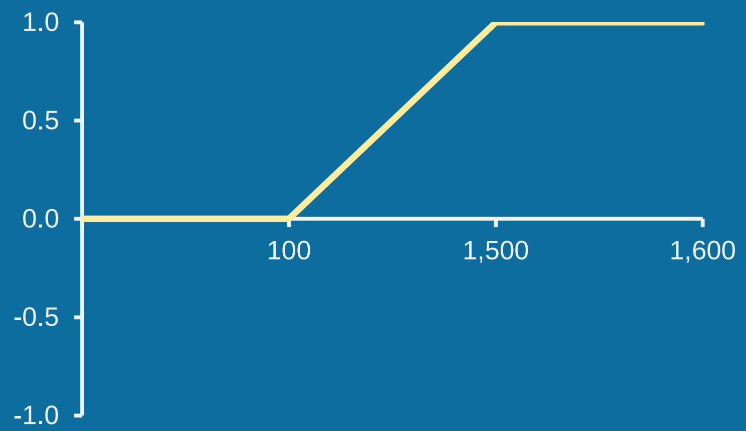
$$((1.0 * F_1 (\text{Estimate of CIN}_{f00} \text{ at t + 1})) +$$
$$(0.1053 * F_2 (\delta_{CIN} |\text{Vertical Shear}| \text{ 975-725 mb at f01})) +$$
$$(0.357 * F_3 (\delta_{CIN}\ d\theta_e/dz \text{ 875-675 mb at f01})))$$

where $F_{\#}$ denotes the appropriate field-specific fuzzy function from the next three slides.

Step 7 of 7

For each level between 1000 and 700 mb, inclusive, threshold the corrected estimates of the next CAPE and CIN analysis fields, and assign grid point values based on whether a thresholded estimate is isolated or is vertically juxtaposed to a similar point. Then, sum the grid point values in each vertical column.

Estimate of $CAPE_{f00}$ at t + 1

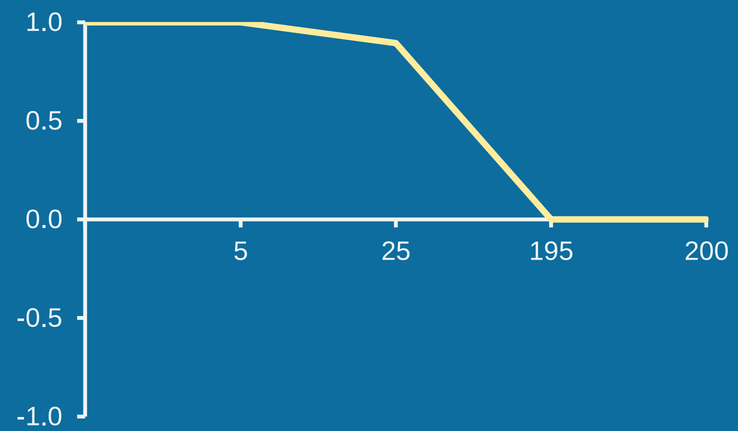


$$y = 0: \ x \leq 100\, J/kg$$

$$y = 0.000\overline{714285}x - 0.0\overline{714285}: \ 100\, J/kg < x < 1500\, J/kg$$

$$y = 1: \ 1500\, J/kg \leq x$$

Estimate of CIN_{f00} at t + 1



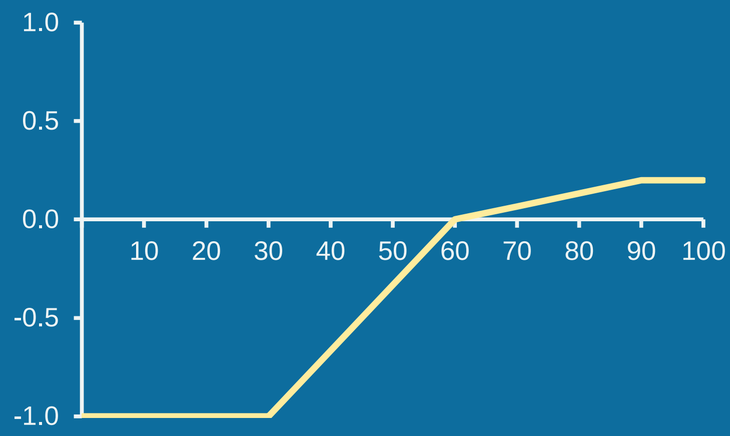
$$y = 1: \ x \leq 5\, J/kg$$

$$y = -0.00525x + 1.02625: \ 5\, J/kg < x \leq 25\, J/kg$$

$$y \approx -0.00526x + 1.02662: \ 25\, J/kg < x < 195\, J/kg$$

$$y = 0: \ 195\, J/kg \leq x$$

δ_{CAPE} Average RH 875-625 mb at f01



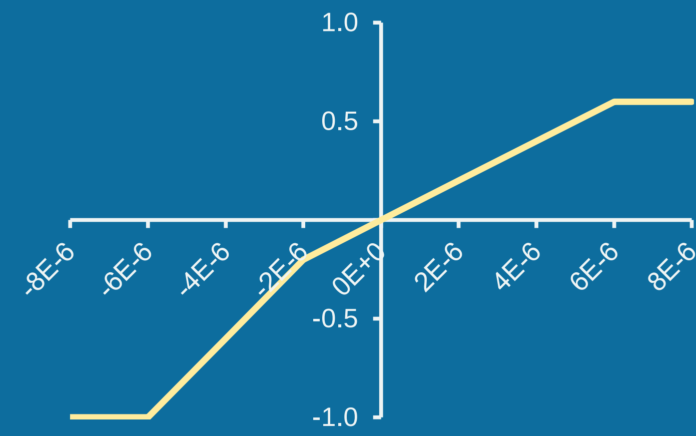
$$y = -1: \ \ 0\% \leq x \leq 30\%$$

$$y = 0.0\overline{3}x - 2: \ \ 30\% < x \leq 60\%$$

$$y = 0.00\overline{6}x - 0.4: \ \ 60\% < x < 90\%$$

$$y = 0.2: \ \ 90\% \leq x \leq 100\%$$

δ_{CAPE} $d\theta_e/dz$ 875-675 mb at f01



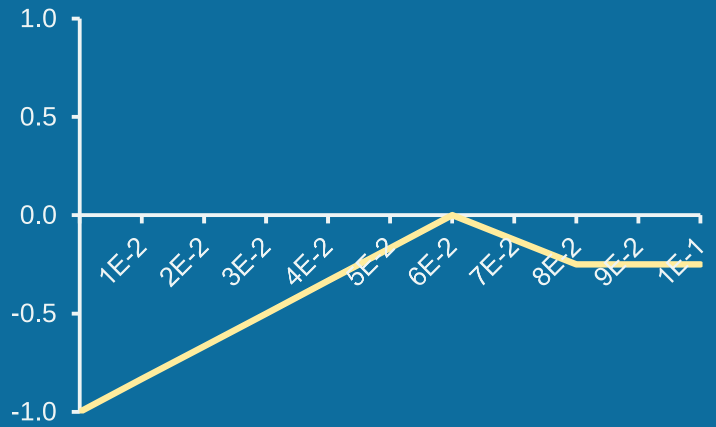
$$y = -1: \ \ x \leq -6e^{-6}\, K/s/mb$$

$$y = 2x + 0.2: \ \ -6e^{-6}\, K/s/mb < x \leq -2e^{-6}\, K/s/mb$$

$$y = x: \ \ -2e^{-6}\, K/s/mb < x < 6e^{-6}\, K/s/mb$$

$$y = 0.6: \ \ 6e^{-6}\, K/s/mb \leq x$$

δ_{CIN} |Vertical Shear| 975-725 mb at f01

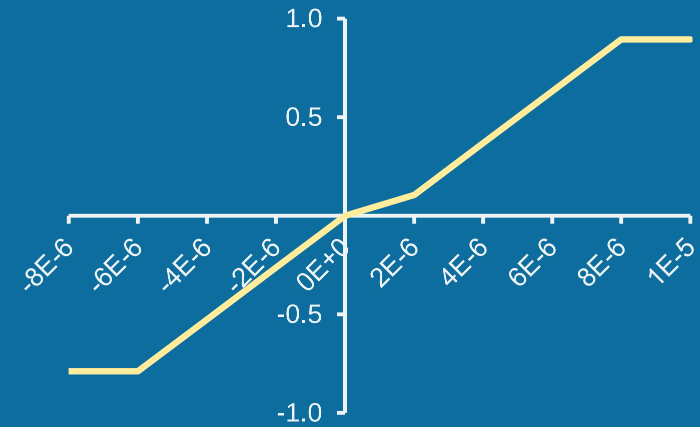


$$y = 0.16x - 1\text{:}\ \ 0\ m/s/mb \le x \le 6e^{-2}\, m/s/mb$$

$$y = -0.125x + 0.75\text{:}\ \ 6e^{-2}\, m/s/mb < x < 8e^{-2}\, m/s/mb$$

$$y = -0.25\text{:}\ \ 8e^{-2}\, m/s/mb \le x$$

δ_{CIN} $d\theta_e/dz$ 875-675 mb at f01



$$y = -0.789\text{:}\ \ x \le -6e^{-6}\, K/s/mb$$

$$y = 0.1315x\text{:}\ \ -6e^{-6}\, K/s/mb < x \le 0\, K/s/mb$$

$$y = 0.05265x\text{:}\ \ 0\, K/s/mb < x \le 2e^{-6}\, K/s/mb$$

$$y = 0.1315\overline{6}x - 0.1578\overline{3}\text{:}\ \ 2e^{-6}\, K/s/mb < x < 8e^{-6}\, K/s/mb$$

$$y = 0.8947\text{:}\ \ 8e^{-6}\, K/s/mb \le x$$

Appendix C: Creating the Likelihood of Cu in 1 hr

Step 1 of 5: Classify

Input both GOES East's and GOES West's visible, near infrared (3.9 microns), water vapor (6.5 microns), and infrared (10.7 microns) data to NRL Monterey's cloud classification algorithm (Bankert et al., 2009). The outputs are stitched together at 105°W to form a single cloud classification grid.

Step 2 of 5: Filter

For each grid point in the cloud classification grid, determine how the cloud classification algorithm classified that point. For points classified as either Cu or CuC, place a +1 in output grid #1. For points classified as clear sky, place a -1 in output grid #1. For all other points, place a 0 in output grid #1.

Step 3 of 5: Expand

For each grid point in output grid #1, search a 17 x 17 (i.e., a 0.34° x 0.34°) grid box centered at that grid point for the maximum value in that grid box, and place that maximum value at that same grid point in output grid #2.

Such expansion is done because ANC "tries to define areas of cloud that work for both day and night (no VIS data at night) but tends to fragment the data." So, it "expands these areas [with Cu and CuC] so it doesn't overestimate the clear areas. Clear areas are weighted heavily in the forecasts. That is, ANC gives a negative weight in areas where there are no clouds, so it [shouldn't be] too aggressive with defining clear areas." (Megenhardt, personal email, 2011)

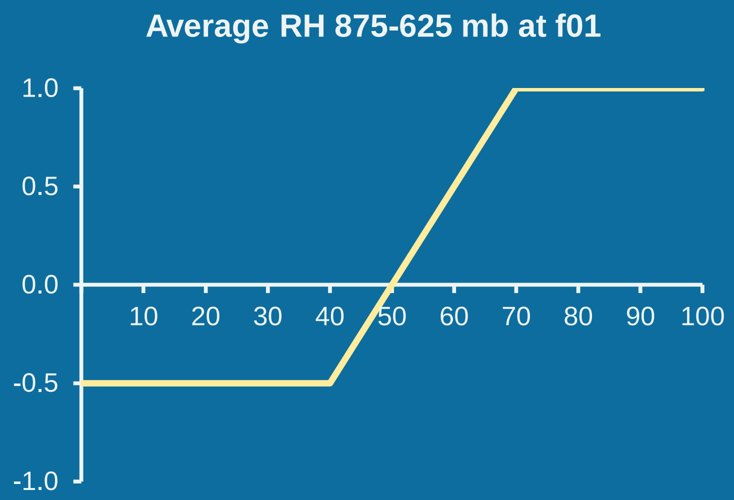
Step 4 of 5: Smooth

For each grid point in output grid #2, and using a 9 x 9 (i.e., a 0.18° x 0.18°) grid box centered at that grid point, calculate the mean value in that grid box, and place that mean value at that same grid point in output grid #3.

Step 5 of 5: Advect

Using the RAP f00 750 mb wind vectors, advect output grid #3 forward in time 60 minutes.

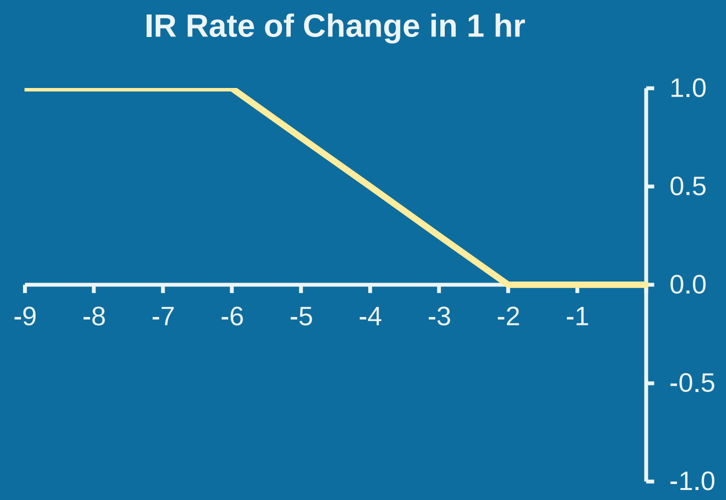
Appendix D: ANC's Predictor-Specific Fuzzy Functions



$y = -0.5$: $0\% \leq x \leq 40\%$

$y = 0.05x - 2.5$: $40\% < x < 70\%$

$y = 1$: $70\% \leq x \leq 100\%$

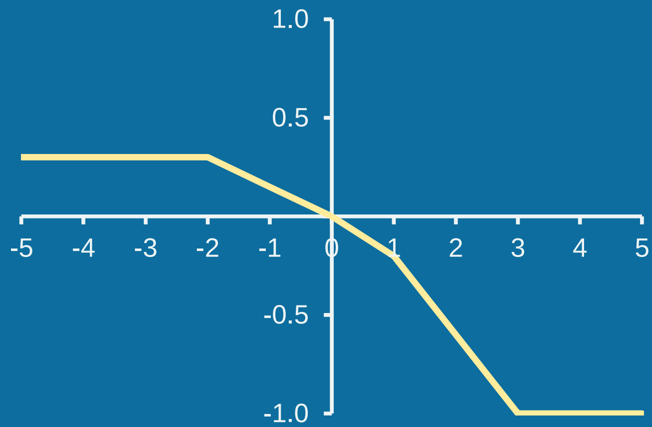


$y = 1$: $x \leq -6\,℃/15\,min$

$y = -0.25x - 0.5$: $-6\,℃/15\,min < x < -2\,℃/15\,min$

$y = 0$: $-2\,℃/15\,min \leq x$

Lifted Index in 1 hr



$y = 0.3$: $x \leq -2\,℃$

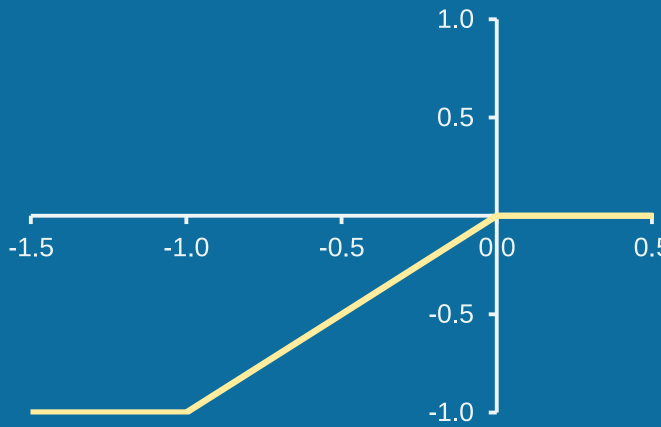
$y = -0.15x$: $-2\,°C < x < 0\,°C$

$y = -0.2x$: $0\,°C \leq x \leq 1\,°C$

$y = -0.4x + 0.2$: $1\,°C < x < 3\,°C$

$y = -1$: $3\,°C \leq x$

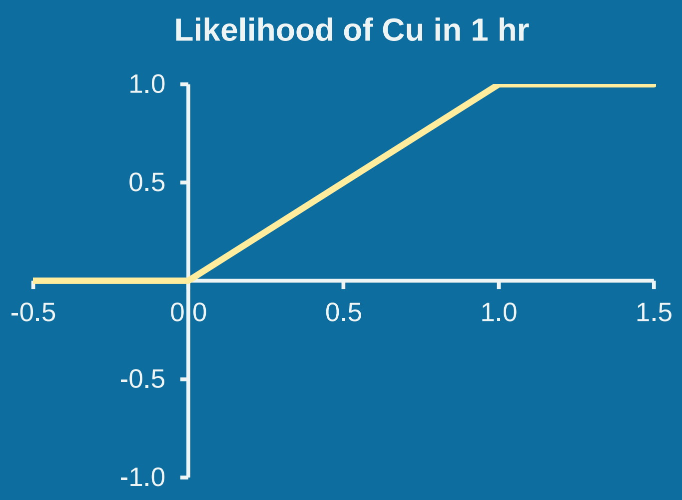
Likelihood of Clear Sky in 1 hr



$y = -1$: $x \leq -1$

$y = x$: $-1 < x < 0$

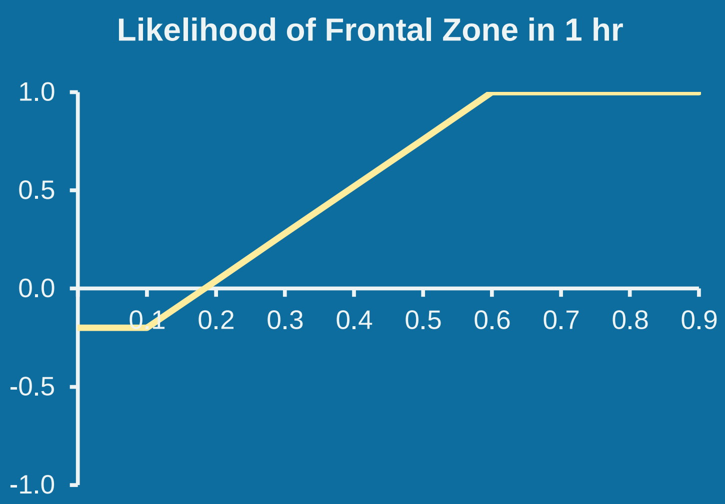
$y = 0$: $0 \leq x$



$$y = 0:\ \ x \leq 0$$

$$y = x:\ \ 0 < x < 1$$

$$y = 1:\ \ x \geq 1$$

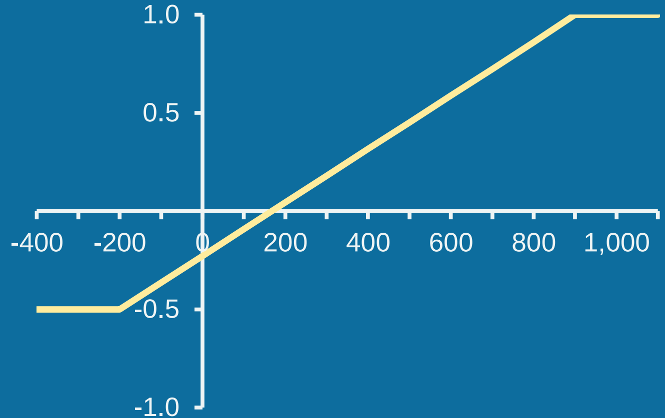


$$y = -0.2:\ \ x \leq 0.1$$

$$y = 2.4x - 0.44:\ \ 0.1 < x < 0.6$$

$$y = 1:\ \ 0.6 \leq x$$

Max CAPE 900-700 mb at f01

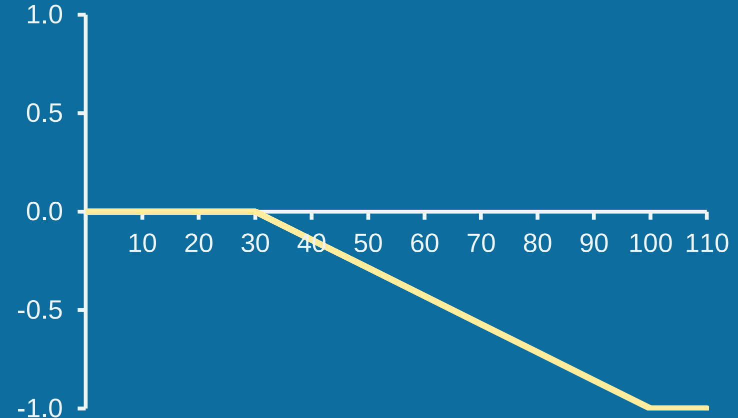


$$y = -0.5: \ \ x \leq -200\, J/kg$$

$$y = 0.001\overline{36}x - 0.2\overline{27}: \ \ -200\, J/kg < x < 900\, J/kg$$

$$y = 1: \ \ 900\, J/kg \leq x$$

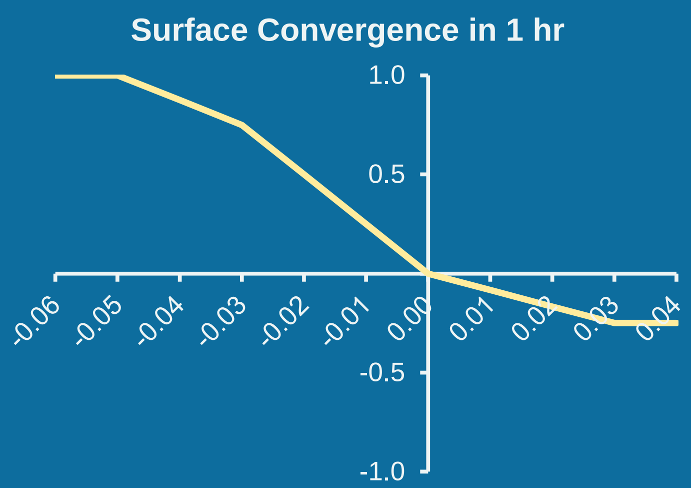
Mean CIN 975-900 mb at f01

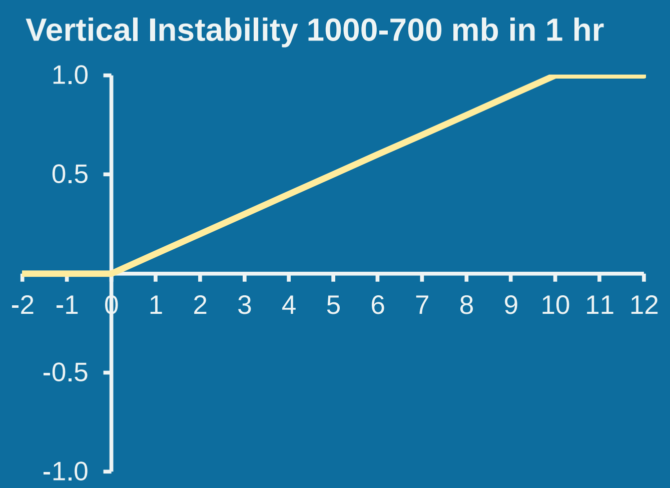


$$y = 0: \ \ x \leq 30\, J/kg$$

$$y = -0.0\overline{142857}x + 0.\overline{428571}: \ \ 30\, J/kg < x < 100\, J/kg$$

$$y = -1: \ \ 100\, J/kg \leq x$$





$$y = 1: \; x \le -0.05\ /10^3 s$$

$$y = -12.5x + 0.375: \; -0.05\ /10^3 s < x < -0.03\ /10^3 s$$

$$y = -25x: \; -0.03\ /10^3 s \le x \le 0\ /10^3 s$$

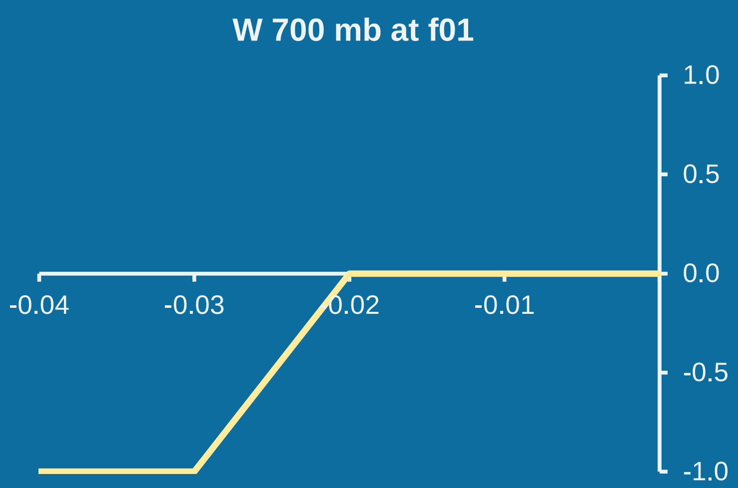
$$y = -8.\overline{3}x: \; 0\ /10^3 s < x < 0.03\ /10^3 s$$

$$y = -0.25: \; 0.03\ /10^3 s \le x$$

$$y = 0: \; x \le 0$$

$$y = 0.1x: \; 0 < x < 10$$

$$y = 1: \; x \ge 10$$



$$y = -1: \quad x \le -0.03\ m/s$$

$$y = 100x + 2: \quad -0.03\ m/s < x < -0.02\ m/s$$

$$y = 0: \quad x \ge -0.02\ m/s$$